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Superstructure of Theme Building of New York World's Fair

BY

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WITH DISCUSSION BY

MESSRS. LOUIS BALOG, AND SHORTRIDGE HARDESTY AND ALFRED HEDEFINE.

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SUPERSTRUCTURE OF THEME BUILDING OF NEW YORK WORLD'S FAIR

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With Discussion by Messrs. Louis Balog, and Shortridge Hardesty and Alfred Hedefine.

Synopsis

There are discussed in this paper the unusual engineering problems involved in the design and construction of the Theme Center of the 1939 New York World's Fair. The points of special interest are the spherical steel framework of the Perisphere, the triangular steel tower of the Trylon, and the determination of the wind loads to which these two structures may be subjected.

Introduction

The Theme Building for the 1939 New York World's Fair, the general features of which are shown in Fig. 1, is composed of four integral parts—the Perisphere, the Trylon, the Bridge, and the Helicline. The Perisphere is the hollow white sphere 180 ft in diameter—as tall as a 16-story building. The Trylon is the needle-like structure—a slender, triangular pyramid taller than the Washington Monument, with its axis 200 ft southeast of the center of the Perisphere. The Bridge is the connecting link between the Perisphere and Trylon, and carries escalator units and walkways. The Helicline is the inclined circular ramp, 950 ft long, which serves as the exit from the group.

The Perisphere, the Trylon, and the Bridge are of steel-frame construction, covered for the most part with a thin stucco covering of a special type, and supported on concrete foundations that rest on creosoted timber piles. The different structures presented individual design problems, some of which were very unusual. The latter statement is particularly true of the Perisphere, which involved the construction of a complete spherical framework, subjected to both vertical loads and transverse wind forces. For its design, the application of the theories of spherical shells was studied thoroughly; but it was

Note.—Published in September, 1940, *Proceedings*.

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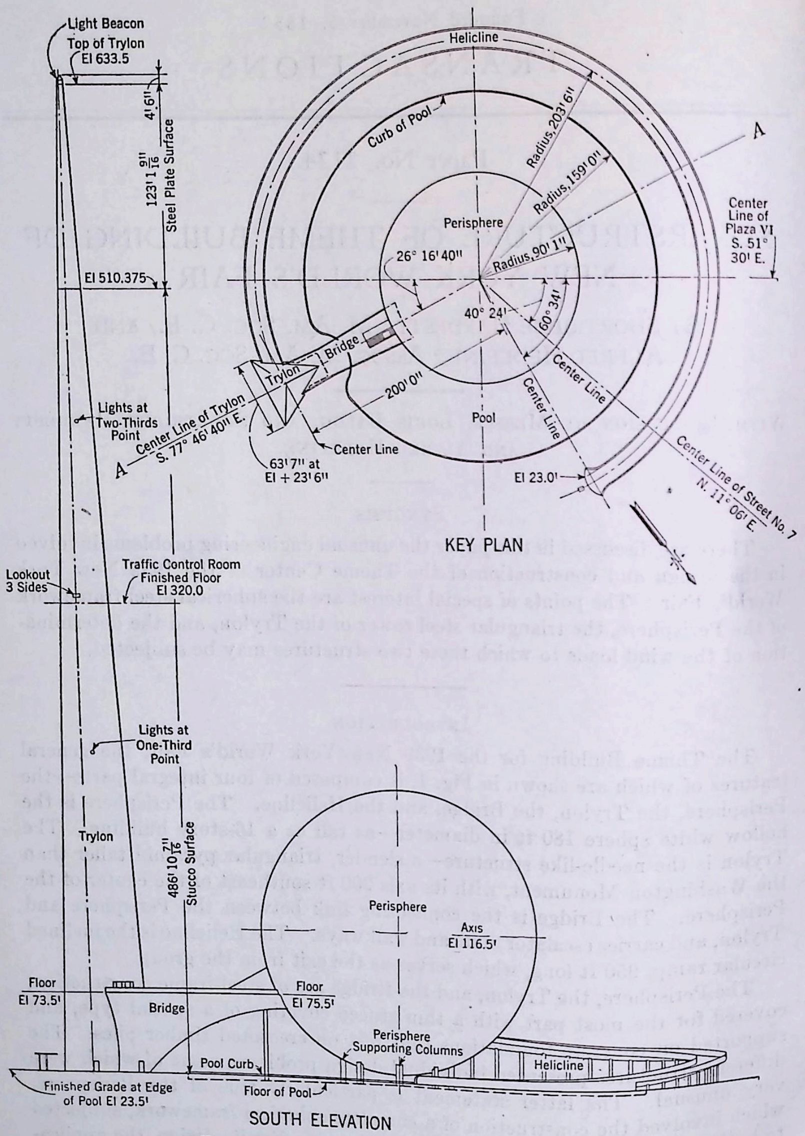


Fig. 1.—Theme Building, New York World's Fair

eventually found possible, by means of careful reasoning and the visualization of the stress action throughout the frame, to employ the methods applied in the analysis of usual structural frames, in spite of the novelty of the arrangement of the members involved.

The Trylon was unique in that it involved the use of a triangular braced tower so slender that the design was controlled by the wind forces rather than the dead and live loads.

Interesting data regarding the four units of the Theme Building are as follows:

Item No.	Description	Value
	Perisphere	
1	Finished outside diameter, in feet..........	179.17
2	Finished height above ground, in feet.......	182.58
	Diameter of Steel Frame, in Feet:	
3	Outer diameter..........................	178.58
4	Inner diameter..........................	162.25
	Meridian Trusses:	
5	Number..................................	32
	Depths, in Feet, at:	
6	Bottom..................................	11.0
7	Equator.................................	8.0
8	Top.....................................	5.0
	Girt Trusses:	
9	Number..................................	15
10	Depths, in feet.........................	5 to 10.5
	Ring Girder at Columns (Feet):	
11	Diameter................................	72.0
12	Depth...................................	7.71
13	Width...................................	2.01
	Columns:	
14	Number..................................	8
15	Height, in feet.........................	16.75
16	Section, in feet........................	2.65 by 1.88
17	Number of pieces........................	6,600
18	Number of shop rivets...................	150,000
19	Number of field rivets..................	100,000
20	Weight of steel, in pounds..............	4,300,000
	Covering:	
21	Total area of covering, in square feet.....	100,851
22	Layers of ½-in. gypsum board (damp-proofed between layers and nailed to wooden struts, 2 in. by 4 in.)..........	2
23	Thickness of stucco, in inches (special magnesite compound)...............	0.25
	Moving Platforms:	
24	Number (an upper and a lower).........	2
	Dimensions, in Feet:	
25	Height above ground (upper and lower)	64, 52
26	Height above Theme exhibit (upper and lower).............	30, 18
27	Diameter to outer edge (upper and lower)..............	113, 103
28	Width............................	6
29	Distance from inner wall of sphere.....	12
30	Weight, each platform, in pounds........	200,000

Item No.	Description	Value
31	Speed, in feet per minute	60
32	Riding time, in minutes	5.5
33	Estimated spectator load, in pounds	120,000
	Dimensions of Reflecting Pool, in Feet:	
34	Diameter	318
35	Depth	1.5 to 3.5
	Foundations:	
36	Length of piles (528 creosoted Douglas fir), in feet	95 to 104
	Footing Dimensions, in Feet:	
37	Diameter	71
38	Width	14.5
39	Depth	3.5
	Ring Wall Dimensions, in Feet:	
40	Diameter	72
41	Width	4.5
42	Depth	7.75
	Dead Load, in Pounds:	
43	Structural framework	4,300,000
44	Outer shell	720,000
45	Inner shell, insulation, walkways, exhibit, and equipment	1,900,000
46	Moving platforms	400,000
47	Subtotal (items 43–46)	7,320,000
48	Live load, in pounds	480,000
49	Snow load, in pounds	370,000
	Total Load, in Pounds:	
50	8 columns (items 47–49)	8,170,000
51	1 column	1,022,000
	Wind Loads, in Pounds per Column:	
52	Transferred load	270,000
53	Shear	82,000
54	Moment at bottom of column due to wind loads, in pound-feet	656,000
	Trylon	
	Dimensions of Superstructure, in Feet:	
55	Height above ground	610.0
56	Base (length of each side)	63.58
57	Top (length of each side)	2.58
58	Distance of observation room (for traffic control) above ground	296.5
59	Weight of steel, in pounds	1,830,000
	Covering:	
60	Total area, in square feet	60,540
61	Year 1939	
	Layers of gypsum board (see item 22)	1
	Outer surface, magnesite compound stucco	....
62	Year 1940	
	Lower 100 ft stucco	....
	390 ft of plywood painted white	....
63	Height of steel plate construction, in feet, from the top	124
	Foundation Dimensions, in Feet:	
64	Length of piles (513 creosoted Douglas fir)	95 to 104

Item No.	Description	Value
	Hexagonal Bases:	
65	Thickness of two 40-ft bases	7.00
66	Thickness of one 46-ft base	7.75
	Dead Load, in Pounds:	
67	Structural framework	1,830,000
68	Covering	270,000
69	Walls, ceilings, enclosures, anchors	1,200,000
70	Subtotal (items 67–69)	3,300,000
71	Live load, in pounds	110,000
	Total Load, in Pounds:	
72	3 columns (items 70–71)	3,410,000
73	1 column	1,137,000
	Wind Load per Column, in Pounds:	
74	Downward	2,580,000
75	Uplift	2,870,000
	Maximum Load per Column, in Pounds:	
76	Downward	3,717,000
77	Uplift	1,770,000
	Bridge	
	Dimensions of Superstructure, in Feet:	
78	Length	98.5
79	Height at Perisphere and at Trylon	52, 50
80	Radius of soffit line	113.72
81	Length of supporting columns at the Perisphere	49
82	Weight of steel, in pounds	290,000
83	Covering, magnesite compound stucco	....
	Moving Stairways (Two, an Upper and a Lower):	
84	Length, in feet (upper and lower)	120, 96
85	Vertical height, in feet (upper and lower)	60, 48
86	Width, each, in feet	2
87	Angle of rise, in degrees	30
88	Speed, in feet per minute	90
89	Capacity in persons per hour, each	8,000
90	Power (electricity)	....
	Foundations:	
91	Number of piles (creosoted Douglas fir)	32
92	Base dimensions, in feet	23.5 by 10 by 5
	Helicline	
93	Grade (percentage)	5.5
	Dimensions, in Feet:	
94	Length	950
95	Width	18
96	Radius	203.5
	Pipe Column Supports:	
97	Height	18 to 50
98	Diameter	1.5 to 2.5
99	Deck material (timber with stainless steel soffit and cork flooring)	....
100	Total cost of Theme Buildings, in dollars	2,000,000

Preliminary Design Considerations: Wind Loads

Before the engineers began the design of the Perisphere and Trylon, the architects had determined the proportions of the two objects and the general arrangement of the passageways, escalators, moving platforms, and exhibits, and had decided that the Perisphere would be over a circular pool, supported on a row of eight columns arranged in a circle of 72-ft diameter, or on a continuous ring of the same diameter. The column supports were finally chosen, partly for architectural reasons, and partly because wind tunnel tests (described subsequently herein) showed that the wind pressure against the Perisphere would be less for the column supports than for the continuous ring. After the framework of the Perisphere and Trylon had been laid out, the details of the passageways, platforms, etc., were determined, and the resulting loads on the frameworks computed. By assuming the type of cover to be used, the vertical loads, both dead and live, on all parts of the two structures could be calculated.

For the determination of the dead loads, the cover was assumed to be cement stucco, 1 in. thick, weighing 17 lb per sq ft. When a light cover, of a patented magnesite compound, was finally selected in 1939, the Perisphere design was not affected; but the reduction of load on the Trylon required that additional weight be provided to overcome the increased uplift from wind loads. This result was secured by supporting the concrete floor of the Trylon at ground level on heavy concrete beams in which the bottom bracing members of the Trylon framework were embedded.

The foundation materials at the site of the Theme Center consist of about 30 ft of cinders and about 45 ft of mud resting on firm material. In line with other heavy foundations on the Fair grounds, piles driven to the firm material were used; and in order to make it safe to leave the structures standing after the closing of the Fair, if desired, creosoted piles were adopted.

It was obvious that the design of the Perisphere and Trylon would be affected by wind loads; and since structures of these types and sizes had not been constructed before, and information relative to the probable wind action on them was scarce and unsatisfactory, it was decided that a wind-tunnel investigation should be made. This investigation consisted of a series of tests performed under the supervision of Alexander Klemin (Chairman of the Department, Daniel Guggenheim School of Aeronautics, College of Engineering, New York University, New York, N. Y.).[3] The Perisphere model was 2 ft in diameter and was raised 0.08 ft above the ground board; the Trylon model was 6.75 ft high, with a triangular base 0.66 ft on a side. All tests were made in a 9-ft wind tunnel at the laboratory.

The test program included drag and pressure tests on the Trylon and Perisphere, alone and in combination with each other, in various positions relative to the direction of the wind. In order that the "ground effects" experienced by the structures as actually built might be reproduced as closely as possible, the tests of the Perisphere, with and without the Trylon, were made with a "ground board" in the tunnel. The kinematic viscosity was assumed to be 0.000154 ft squared per sec, and the Reynolds number was taken equal to the product of the wind speed in feet per second by the diameter in feet by the constant 6,500 for the case of the Perisphere, and by the length in feet of

[3] "Aerodynamics of the Perisphere and Trylon at World's Fair," by A. Klemin, E. B. Schaefer, and J. G. Beerer, Jr., *Proceedings*, Am. Soc. C. E., May, 1938, p. 887.

a side of the base by the same constant for the case of the Trylon. The largest values of the Reynolds number for the wind-tunnel tests, for the cases of the Trylon alone, the Perisphere alone, and the Perisphere and Trylon in combination were 378,000, 572,000, and 1,340,000, respectively.

Table 1 indicates the values of the pressures on the Perisphere and Trylon for the various conditions tested. It shows the pressure, in pounds per square

TABLE 1.—PRESSURES IN POUNDS PER SQUARE FOOT OF PROJECTED AREA

Item No.	Description	Formula[a]	Shape factor	PRESSURE[a] FOR: $V = 100$	$V = 90$
1	Velocity head	$0.00256\ V^2$	1.0	25.6	20.8
2	Large, square, flat plate, normal to wind	$0.0032\ V^2$	1.25	32	26
3	Prism 1 by 1 by 3	$0.0038\ V^2$	1.48	38	30.8
4	Infinitely long prism or flat plate	$0.0050\ V^2$	1.95	50	40.5
5	Trylon on the ground: Flat side into wind	$0.0037\ V^2$	1.45	37	30
6	Edge into wind	$0.0023\ V^2$	0.90	23	18.6
7	Perisphere alone, on ring wall, on the ground	$0.0015\ V^2$	0.59	15	12.2
8	Perisphere on ring wall, with Trylon, on ground	$0.0018\ V^2$	0.70	18	14.6
9	Perisphere alone, on columns, on ground	$0.0013\ V^2$	0.51	13	10.5
10	Perisphere on columns, with Trylon, on ground	$0.0016\ V^2$	0.62	16	13
11	Perisphere in free air	$0.0005\ V^2$	0.195	5	4

[a] V = velocity in miles per hour.

foot of projected area, for any velocity in miles per hour, together with the values for 100-mile and 90-mile velocities. For reference, the table also includes values for velocity head, and for pressures on large square flat plates, on prisms three times as long as wide, and on indefinitely long prisms or flat plates.

Current specifications usually provide that bridges and buildings shall be designed for a maximum wind pressure of 30 lb per sq ft. By referring to Table 1 it will be noted that this pressure corresponds to the value for a large square flat plate at a wind velocity of 100 miles per hr, and also to that for a 1 by 1 by 3 prism in a 90-mile wind.

The "shape factors" determined from the tests on the Trylon and Perisphere, expressed as the ratio of the average wind pressure to the velocity head, are worthy of note. The free Perisphere in an air flow of 90 miles per hr gave an average pressure of 4 lb per sq ft on the projected area, corresponding to a shape factor of 0.195; whereas, when the Perisphere was subject to the same air flow in the presence of the ground, supporting columns, and Trylon, the shape factor increased to 0.62, corresponding to an average pressure of 13 lb per sq ft on the projected area, or an increase in the wind force of three and a quarter times. This phenomenon, caused by the ground and structure effects, indicates the importance of the tests from the structural design point of view.

It is of interest to note that, in the test of the Trylon with a flat side into the wind, there were pressure decreases, or suctions, of sufficient force on those faces away from the direction of the wind to result in a force greater than the velocity pressure times the projected area, corresponding to a shape factor of 1.45. It might also be mentioned that tests with the Trylon directly in front

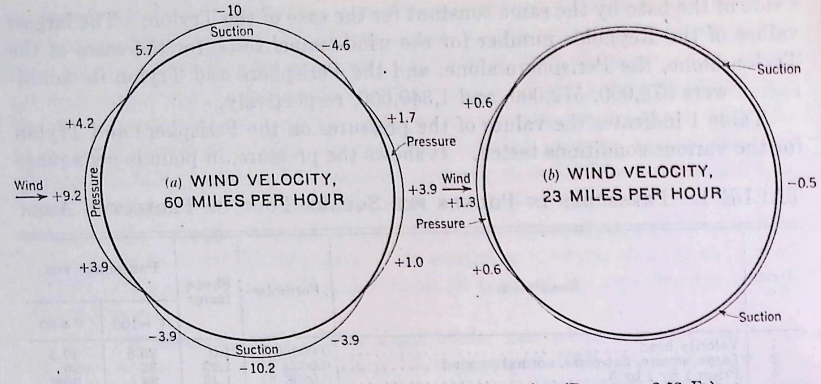


FIG. 2.—WIND PRESSURES ON A SPHERE IN FREE AIR (DIAMETER 9.52 FT)

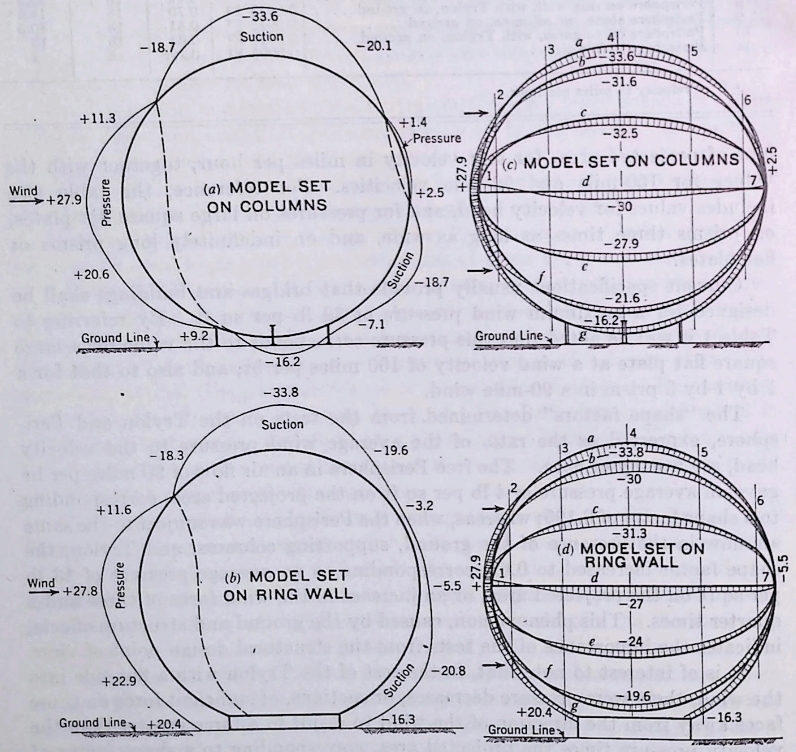


FIG. 3.—PRESSURES ON A FREE PERISPHERE MODEL DUE TO A WIND VELOCITY OF 100 MILES PER HR

of the Perisphere, blocking the path of the wind, gave a net pressure on the Perisphere of practically nothing.

The values in Table 1 indicate the horizontal resultant of the pressures created by the flow of wind. The drag tests on the Perisphere showed, in addition to the horizontal force, a considerable uplift, amounting to 70% of the horizontal force in the case of the Perisphere alone on ring wall adjacent to the ground, and 60% of the horizontal force in the case of the Perisphere alone on columns adjacent to the ground.

Fig. 2 shows the center meridian pressures, in pounds per square foot, for the supercritical and subcritical states, as determined by O. Flachsbart,[4] at the Göttingen Aerodynamics Institute, on a wood sphere 24.2 cm (9.52 in.) in diameter in free air—that is, with no ground board. Figs. 3(*a*) and 3(*c*) indicate the pressures, in pounds per square foot, on the center meridian of the Perisphere for a wind velocity of 100 miles per hr, for the case of the sphere on columns. The broken lines are isobars of zero pressure. Figs. 3(*b*) and 3(*d*) show graphically the pressure contours for the same model, set on a ring wall. The shaded portion on the outside of a great circle indicates positive pressure, whereas that on the inside of the circle represents a negative pressure, or suction.

The diagrams of Fig. 4 represent the pressures resulting from a wind velocity of 100 miles per hr, based on the various cases tested by Professor Klemin[3] (see Fig. 5). Case I is for that of the Perisphere alone on columns. In this case it is of interest to note that a turbulence was created in the flow of air tailing off the rear of the sphere, thereby forming a back pressure acting against the direction of the wind. Here, as in all cases, the maximum positive pressure occurs on the surface exposed directly to the wind, and the maximum negative pressure on the areas at right angles to the direction of the wind. Case II is for the condition of the Perisphere on columns with the Trylon at its side (with respect to the direction of the wind). This gives essentially the same family of pressure curves as in Case I. The back pressure of Case I, however, has now changed to a slight suction. Case III is for the Perisphere on a ring wall. For Case IV there is the same combination as Case II except that the Perisphere is on a ring wall instead of columns. In both Case III and Case IV there is a considerable build-up of suction at the rear of the sphere, which increases measurably the net force tending to push the Perisphere off of its foundations. Case V is that of the Perisphere on the ring wall with the Trylon directly in front of it with respect to the wind direction. The net result of the pressures for this last case, for all practical purposes, was negligible.

The pressures plotted for each case are those along the seven semi-arcs of great circles which pass through the front and rear poles and whose planes make angles above and below the equator of 0°, 30°, 60°, and 90°. The pressure points are located by increments of 30° azimuth angles through 180° from front to rear. The net wind force acting on the Perisphere for each case tested can be obtained by graphic integration of these pressure curves.

The pressures given in the various diagrams represent the net pressures

[4] "Recent Researches on the Air Resistance of Spheres," by O. Flachsbart, *Technical Memorandum No. 475*, National Advisory Committee for Aeronautics, 1928.

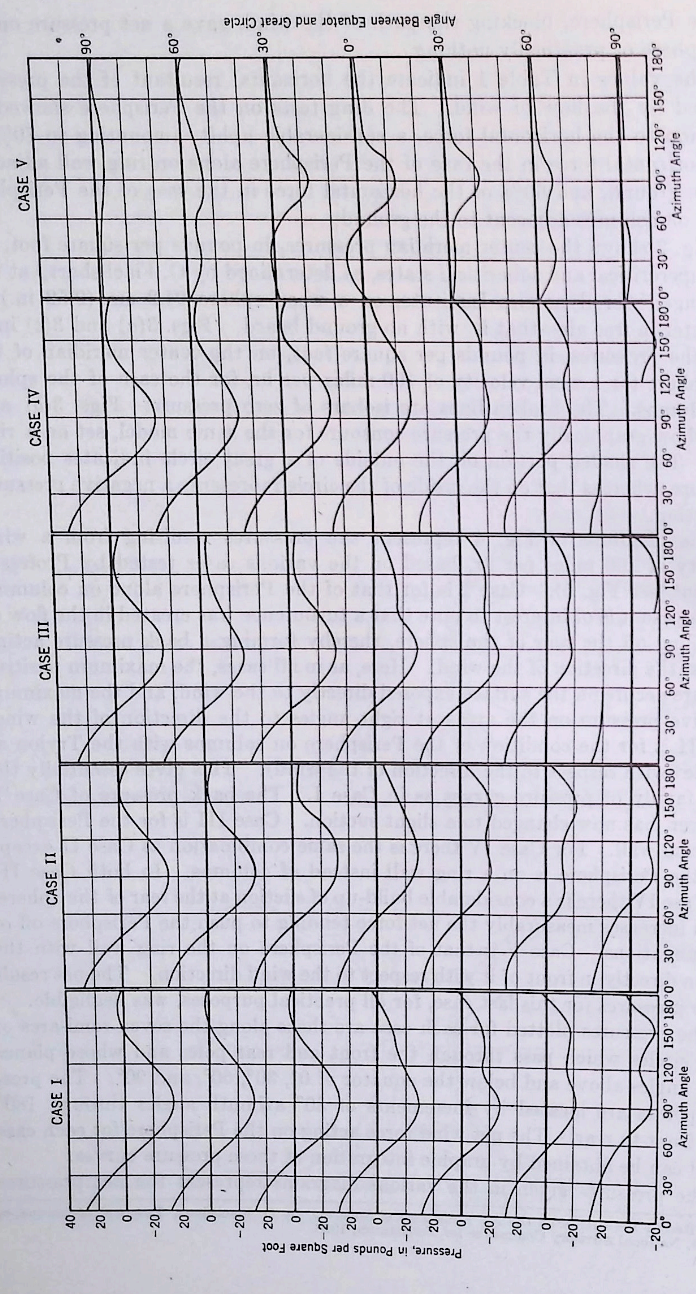


Fig. 4.—Pressures on a Perisphere Model, with Trylon in Various Positions; Wind Velocity 100 Miles per Hr

existing on the Perisphere covering at any point when the air pressure within the Perisphere is the same as that in the moving air on the outside. Since the covering is nearly impervious, however, the pressure within may at times be practically that of still air, which is greater than that of moving air by the amount of velocity head. At such times the net pressures on the covering, instead of varying from the velocity head at the portion facing the wind to a suction about one third greater at a point 180° thereto, will vary from about zero at the first point to a suction two and one third times the velocity head at the second point. The suction acting on the cover, therefore, may be as great as 60 lb per sq ft.

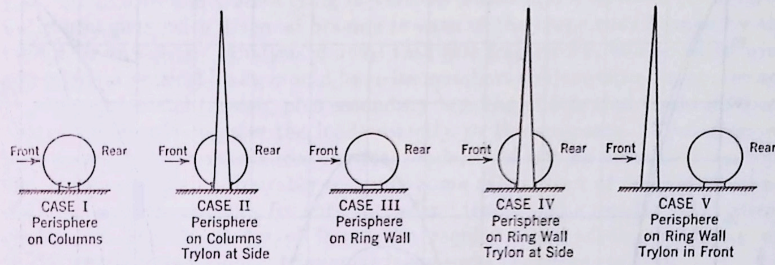


Fig. 5.—Positions of Structures with Respect to Wind, in Tests of Fig. 4

The curves shown in Fig. 6 present an interesting comparison of wind pressures acting on the Perisphere under various conditions.

As has been previously indicated, there is a large variation in the wind pressures acting over the surfaces of the Perisphere and Trylon. For the Trylon there is positive pressure over the face or faces exposed to the wind, and suction on those away from the direction of the wind, with the maximum pressure approximately equal to the velocity head, and the resultant force acting at a point one third of the way up from the ground surface. In the case of the Perisphere, a bulb of positive pressure occurs over the portion of the surface directly facing the wind, and a high negative pressure or suction occurs over the surface area at 90° to the direction of the wind, reducing to a minimum pressure at a point 180° from the wind direction.

In order to arrive at pressure values for the design of the Perisphere, consideration had to be given to the fact that the presence of the Bridge increased appreciably the forces on the Perisphere itself. The Bridge also causes a slight increase in the wind pressure on the Trylon, but this effect is inappreciable, as the force is applied close to the bottom of the Trylon.

The symmetrical area of suction around the complete Perisphere produces tension in that group of meridian trusses, with very little distortion of the girts. The pressure on the front of the Perisphere, the resultant of which is inclined upward, and the suction on the back, the resultant of which is inclined downward, form a set of forces tending to roll the sphere as a whole.

As the maximum wind velocity in the New York area approximates 90 miles per hr, it was decided to base the design loads on that velocity, with higher values for checking stability against overturning. Therefore, the following loads, in pounds per square foot, were adopted:

Description	For designing members	For determining stability and anchorage
Trylon with flat side into wind........	30	50
Trylon with edge into wind...........	20	33
Perisphere on columns..............	15	25

Since the completion of the Perisphere and Trylon, they have been subjected to several heavy winds and found to be satisfactory. During the hurri-

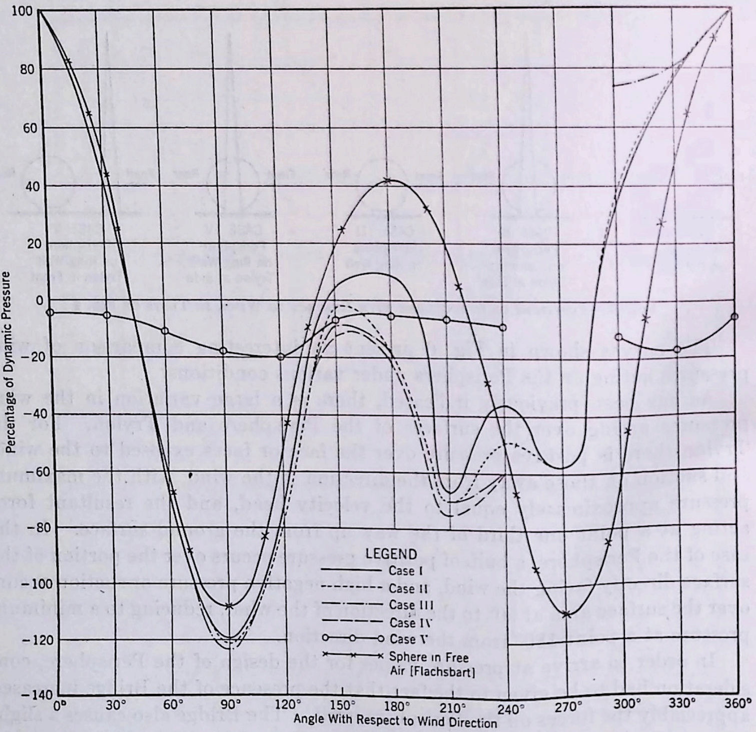


Fig. 6.—Pressures on a Meridian Circle Expressed as Percentages of Dynamic Pressure

cane of September, 1938, a wind velocity of about 75 miles per hr was reached at the site of the Fair. At this time the steel framework of the Trylon, including the steel plate section at the top (but no other outside covering), was completed, and the steel frame of the Perisphere was finished but free of covering or scaffolding. During the storm it was observed that the top of the Trylon moved about 6 in. in either direction from the vertical, which movement compares with the computed movement under full design wind load of about 3 ft.

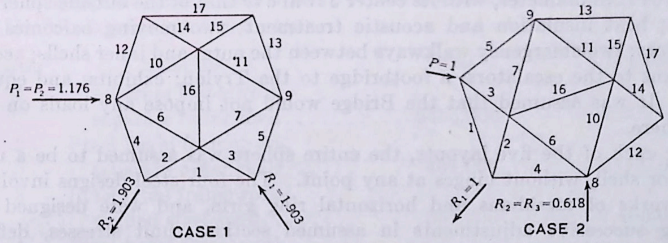
A second observation was made on the Trylon during a subsequent wind which attained a velocity of 45 miles per hr. At that time three fourths of the surface had been covered and a complete scaffolding system shrouded the entire structure. The observed movement at an angle of about 45° to the direction of the wind was about 7 in. on either side of the vertical.

No measurements have been made on the movements of the Perisphere, but after its completion an observer standing at the junction of the Bridge and the Perisphere, with one foot on each structure, noted that only the faintest tremor could be detected during a 45-mile wind.

The framework of the Perisphere was planned to consist of a series of great-circle meridian trusses lying in vertical planes and a series of small-circle horizontal girts, with diagonal bracing in each of the trapezoids formed by the meridians and girts. Analysis showed that this framework, when loaded with symmetrical vertical loads, would have its members subjected to direct stresses of compression and tension, plus secondary bending effects that could be computed, and would transfer the loads directly to the supports. The effect of horizontal forces or unbalanced vertical loads could not be visualized readily, and it was considered desirable to study some other form of framework that could be analyzed directly for such forces and loads. The regular polyhedron having the largest number of faces—the regular icosahedron, consisting of thirty members and twenty triangular faces—was therefore studied.

The icosahedron frame is shown in Table 2. Two loading conditions were considered. In Case 1, the icosahedron is supported at the two ends of one

TABLE 2.—Stresses in the Icosahedron Frame



Number	Stress Influence Values		Number	Stress Influence Values		Number	Stress Influence Values		Number	Stress Influence Values	
	Case 1	Case 2		Case 1	Case 2		Case 1	Case 2		Case 1	Case 2
1	−0.138	−0.072	6	−1.100	−0.578	11	−0.292	−0.153	16	+0.361	+0.190
2	+1.421	+0.746	7	+0.516	+0.271	12	−0.043	−0.023	17	−0 138	−0.072
3	−1.198	−0.629	8	+0.642	+0.337	13	−0.043	−0.023	....	....	....
4	+0.458	+0.240	9	+0.362	+0.190	14	+0.112	+0.059	....	....	....
5	−0.543	−0.285	10	−0.292	−0.153	15	+0.112	+0.059	....	....	....

side, and subjected to two horizontal radial forces at mid-height; and in Case 2, it is supported on the three corners of one face, and subjected to a single inclined force. The resulting stresses, which are determinate, are given in the table. By analogy, it was determined that the spherical framework would also transmit transverse forces to the supports by means of direct stresses in the frame members plus secondary bending effects, as in the case of vertical loads.

The icosahedron analogy also afforded a means of estimating the wind stresses in the diagonal bracing. For the Perisphere, the horizontal force of $2P$ for Case 1 would be about 330,000 lb, which would produce a compression of about 200,000 lb in the diagonals leading down to the leeward support, and a tension of about 220,000 lb in the diagonals leading down to the windward support. Four diagonal members were available to take each of these forces.

Types of Construction Considered

Perisphere.—Preliminary studies for the Perisphere showed that the following types of construction merited consideration: (1) Steel truss framework, separate outer shell; (2) steel beam framework, separate outer shell; (3) steel truss framework, welded steel shell; (4) steel beam framework, welded steel shell; and (5) reinforced concrete shell. The following description of these five types is adapted from reports submitted to the Construction Department of the Fair on April 12, 1937:

Each design provided a sphere of 180 ft outside diameter, carried on eight columns resting on concrete foundations. The columns were to be on a circle of 36-ft radius, the bottom of the sphere being placed 3 ft above the surface of the pool. It was assumed that the outer surface was to be watertight, of accurate outline, as smooth as practicable, and preferably free from visible joints. Provision was made in each design for supporting a lighter inner shell about 164 ft in diameter, with its center 3 ft above that of the outside spherical surface; heat insulation and acoustic treatment; two moving balconies for spectators; two emergency walkways between the outer and inner shells; access platforms to the escalators; a footbridge to the Trylon; exhibits; and equipment. It was assumed that the Bridge would not impose any loads on the Perisphere.

For each of the five layouts, the entire sphere was assumed to be a unit frame or shell, without hinges at any point. The four steel designs involved frameworks of meridians and horizontal ring girts, and were designed by making successive adjustments in assumed sections, unit stresses, deflections, and total stresses. The reinforced concrete layout involved a shell, unstiffened except between the columns, and was designed by means of theories developed for spherical shells.

In the first layout, the framework consisted of trussed members from 5 ft to 11 ft deep, occupying the entire space between the outer and inner shells except at the bottom. There were thirty-two main meridians, spaced about 18 ft apart at the equator. In general, the ring girts were about 16.5 ft on centers. The panels between meridians and rings were braced with X-bracing of single-angle members. The outer shell, which was not intended to take stress, was assumed to be carried by vertical purlins supported by the girts. The inner shell was to be carried by light members supported on the inner flanges of the meridians and girts. The meridians were carried by a circular box girder of 36-ft radius, resting on eight supporting columns. The columns were assumed fixed at their upper ends by the ring girder and the meridians. The design assumed riveted construction throughout, although welded work could be used in certain parts if preferred.

The second layout was similar to the first, except that the members in the upper part of the sphere were curved wide-flanged beams instead of trussed members. Hangers would be required to support the inner shell in this part of the sphere.

The third layout involved a framework similar to that of the first layout, but considerably lighter, with a butt-welded steel outer shell that participated in carrying stress. The diagonal bracing was omitted, as the shell performed its function. The shell was $\frac{5}{16}$ in. thick for the upper parts of the sphere, and reached a maximum thickness of $\frac{7}{16}$ in. in the lower part.

The fourth layout was similar to the third, except that the framework in the upper part consisted of curved beams, as in the case of the second layout.

In the reinforced concrete shell layout, the thicknesses as designed were 3 in. at the top, 4 in. at the equator, and 2 ft 6 in. at the supports. Concrete stresses were low, averaging 200 lb per sq in. except at the supports, where large bending moments occurred. It was assumed that the thin parts of the shell would be of gunite, and the thicker parts of poured concrete. There was ample precedent in European practice for the shell thicknesses adopted, but the requirements of American practice might have called for some modifications; and the conditions at the support were sufficiently unusual to have justified the making of model tests for this part if the reinforced concrete design had been adopted.

The estimated costs of the superstructure and the loads on the foundations for the five layouts were as follows:

Layout	Description	Cost	Total load on foundations, in pounds
1	Steel truss framework, separate outer shell..........	$453,000	9,300,000
2	Steel beam framework, separate outer shell..........	401,000	9,000,000
3	Steel truss framework, welded steel shell............	554,000	7,400,000
4	Steel beam framework, welded steel shell............	518,000	7,200,000
5	Reinforced concrete shell........	425,000	15,100,000

The costs of the foundations were to be added in order to determine the comparative total costs.

In comparing the various designs, the following points had to be considered, in addition to the costs:

(1) *Steel Truss Framework, Separate Outer Shell.*—The framework could be fabricated and erected readily by methods in common use, and presented no difficulties other than those arising from the novel shape of the structure. It would provide a stiff, sturdy structure of accurate outline. The outer shell

probably would call for unusual care in construction, if a satisfactory surface were to be obtained. There would be some possibility of objectionable cracks developing as a result of temperature changes or participation in stress action.

(2) *Steel Beam Framework, Separate Outer Shell.*—The comments in item (1) would apply in general to this design. The fabrication should be cheaper than for the first layout, and the erection somewhat more difficult and possibly more expensive. The resulting structure would be entirely satisfactory, although not quite as stiff as in the case of the first layout.

(3) *Steel Truss Framework, Welded Steel Shell.*—The framework members could be fabricated and erected readily. The construction of the shell would call for unusual care in order to secure a satisfactory surface and avoid locked-up temperature stresses. The major difficulty would be to secure even joints; but it was believed that this could be overcome by accurate forming of the plates, and properly controlled welding procedure, with small welding rods. It might be desirable to have tests made to demonstrate the possibilities in this respect. The resulting structure would be somewhat stiffer than that of the first layout; and the shell would be certain to be watertight and free from cracks.

(4) *Steel Beam Framework, Welded Steel Shell.*—The comments in items (1) to (3) would apply in general to item (4), with the modifications noted in connection with layout 2.

(5) *Reinforced Concrete Shell.*—This layout probably would present more construction problems than would the four steel designs, items (1) to (4), but it was believed that they could be solved successfully by companies experienced in concrete construction. The falsework would have to be designed and constructed carefully, in order to secure correct outlines. Accurate workmanship would be required in order to procure a satisfactory outer surface, but this requirement might apply equally to layouts (1) and (2).

After consideration of the construction problems and the relative substructure costs for the various types, it was decided to adopt layout 1, which was found to be no more expensive than the other types, and could be expected to be free from difficult, unforeseen problems that might prove embarrassing. Actual construction has indicated that a correct choice was made.

Trylon.—For the Trylon, four designs were considered, as follows:

Design	Description	Estimated cost of superstructure
1	Braced steel tower construction, surface of thin steel plate	$231,000
2	Braced steel tower construction, gunite or stucco surface	231,000
3	Flat plate construction, consisting of welded steel plate properly stiffened	247,000
4	Reinforced concrete construction	185,000

For designs 1 and 2, the top quarter was to be of flat plate construction similar to design 3.

After taking into consideration the construction problems and the total costs, including foundations, design 2 was adopted.

Geometry and Framework

The outside surface of the completed Perisphere is a true spherical surface, 180 ft in diameter. The inner surface is fundamentally a sphere of 162-ft diameter, with its center 3 ft above the center of the outside sphere; but this surface is modified in the lower part to suit the exhibition and the moving platforms.

The entire Perisphere frame is a unit, of riveted construction throughout, without hinges at any point. The framework consists of curved truss members from 5 ft to 11 ft in depth. There are 32 meridians, each a half of a great circle, intersecting at the top and the bottom, and 15 horizontal girts from 7 ft to 16 ft on centers. The trapezoidal panels formed by the meridians and girts are braced, in the outer surface only, with X-bracing of single-angle members. The intersections of the girts with the meridians lie on small circles; but each section of girt between meridians lies in a great circle. As a result, all meridian and girt members are plane, with all outside chords bent to a

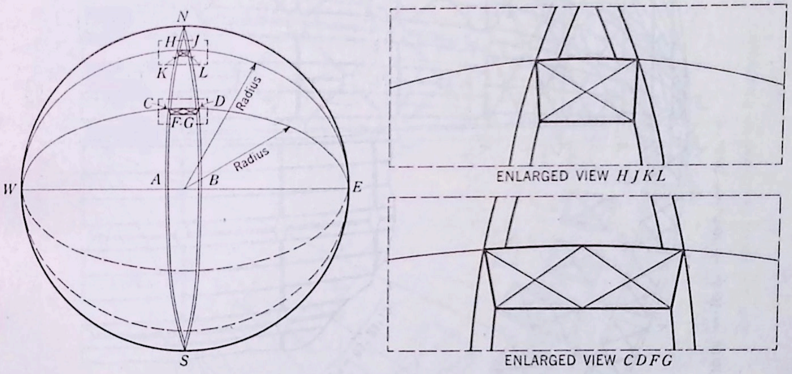
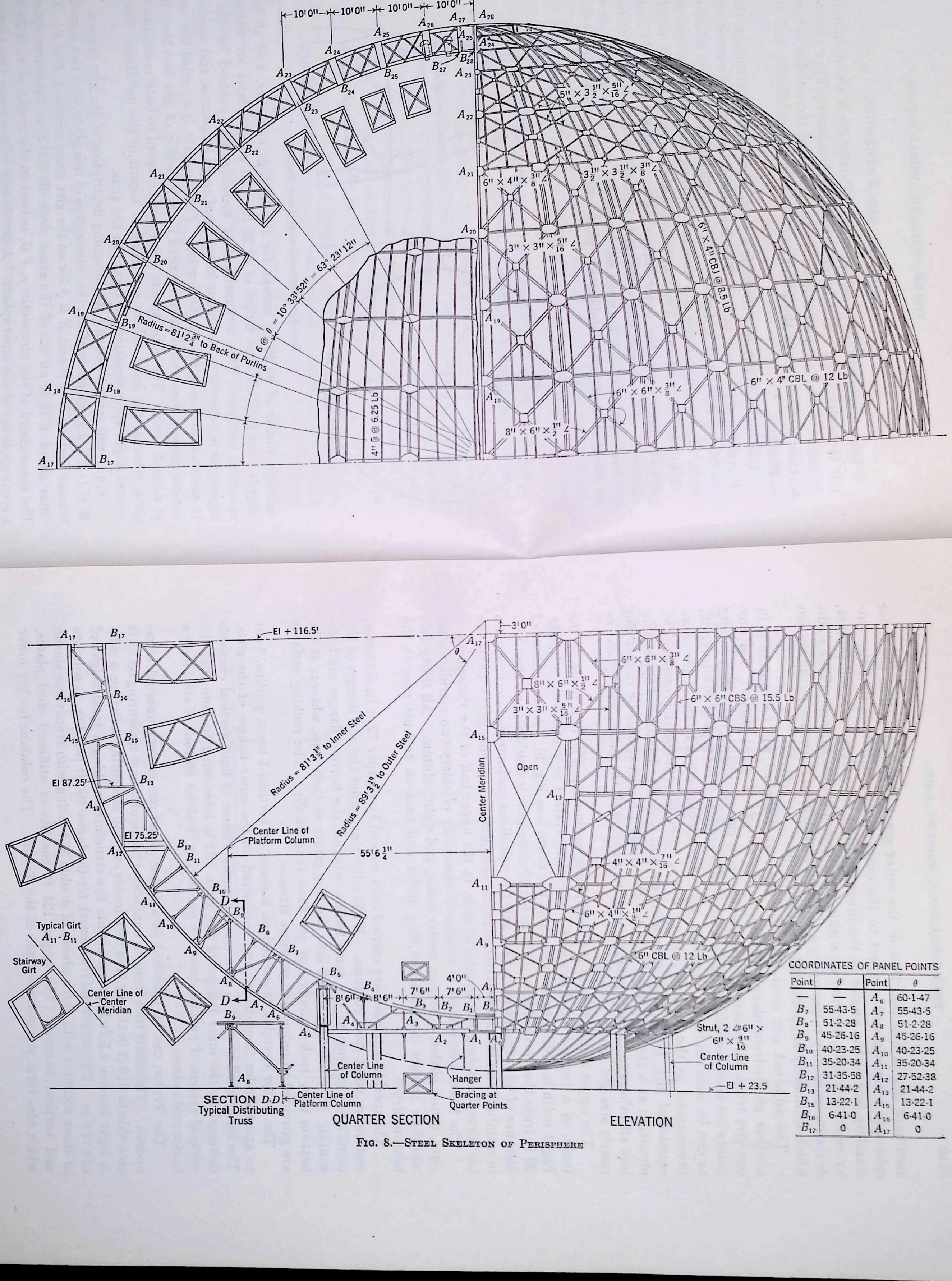


Fig. 7.—Description of Girt Trusses

uniform radius of 89.29 ft, except for the bottom members, and all inside chords bent to a uniform radius of 81.29 ft. All gusset plates are dished to spherical surfaces. This arrangement of members was evolved by the designers; it is believed to be unique in spherical space structures, and conducive to accurate and satisfactory fabrication. For the girts, it avoided the use of conical girders that would have resulted if each section of girt had been bent to a small circle. The outer sphere was exact in regard to both meridians and girts, whereas the inner sphere was exact in regard to the meridians, but involved slight approximations in regard to the girts.

Fig. 7 will serve to clarify the foregoing description of the girt trusses. If $W\,C\,D\,E$ is a great-circle plane, the intersection of this plane with two meridian trusses such as $N\,A\,S$ and $N\,B\,S$ forms the girt truss section $C\,D\,G\,F$. The outer chord, $C\,D$, is an arc of the great circle $W\,C\,D\,E$ of the outer sphere. The inner chord of the girt sections, $F\,G$, is bent to the radius of 81 ft $3\frac{1}{2}$ in., a great circle arc of the inner sphere. A girt section between any two adjacent



COORDINATES OF PANEL POINTS

Point	θ	Point	θ
—	—	A_6	60-1-47
B_7	55-43-5	A_7	55-43-5
B_8	51-2-28	A_8	51-2-28
B_9	45-26-16	A_9	45-26-16
B_{10}	40-23-25	A_{10}	40-23-25
B_{11}	35-20-34	A_{11}	35-20-34
B_{12}	31-35-58	A_{12}	27-52-38
B_{13}	21-44-2	A_{13}	21-44-2
B_{15}	13-22-1	A_{15}	13-22-1
B_{16}	6-41-0	A_{16}	6-41-0
B_{17}	0	A_{17}	0

FIG. 8.—STEEL SKELETON OF PERISPHERE

meridians lies in a plane, that of the outer great circle; and since the inner sphere is eccentric with respect to the outer sphere, it follows that the outstanding legs of the angles of the inner chord of the girt section will project, by small amounts, beyond or within the inner spherical surface. This slight deviation is readily taken up by a minor adjustment in the inside purlin system which ultimately supports the inner covering.

The adjacent girt section to the one just described will be contained in another great-circle plane with its chords as arcs of great circles. For any one girt truss, all points of intersection with the meridians, or corresponding points between meridians, lie on a circle of latitude. This type of construction, as can be seen readily in elevation, gives a scalloping effect to the girts—more pronounced as the distance, up or down, from the equator increases. The equatorial girt, being both a great circle and a circle of latitude, appears in elevation as a straight line. In plan the equatorial girt chords lie in a circle; but all others (and increasingly so as they approach the poles) will appear as nearly straight lines.

The upper hemisphere of the Perisphere, as shown in Fig. 8, is made, in the main, of a top center drum and a complete system of meridians and girts. Eight main meridian trusses begin at the drum (see Fig. 9). At a distance of 20 ft from the vertical axis the number of meridians is doubled; and then at a distance of 40 ft from the vertical axis the meridians are increased to thirty-two in number; and from this latter point on the number is unchanged. There are ten girts, including the equatorial ring, in the upper half of the Perisphere.

The lower hemisphere is composed essentially of thirty-two meridians, six girts, a ring girder, a bottom center drum, and eight columns. All thirty-two meridian trusses intersect the ring girder and pass on to butt against the bottom drum.

The meridian trusses are carried on a curved box girder 72 ft in diameter, resting on eight steel columns. Within the ring girder the top flanges of the meridians continue at the same radius, whereas the bottom flanges are horizontal. The columns frame into the ring girder at their tops, whereas at the bottom they rock radially but are fixed tangentially. The rocking surfaces are provided to reduce temperature stresses.

The outer covering is carried on a series of vertical I-beam purlins, bent to great circles (meridians) and supported on the outer flanges of the girts. The inner shell is carried on a similar purlin system of light vertical channels fastened to the inner flanges of the girts. It is interesting to note that the vertical system of purlins showed a net saving of about 80 tons of steel over a horizontal system.

The Trylon can be classified as a tower structure—a very unusual tetrahedron. This shaft rises from an equilateral triangular base measuring 63.58 ft on a side to 610 ft in the air, and terminates in a top measuring 2.58 ft on a side. A light beacon 4.5 ft high surmounts the structure, making a total height of 614 ft 6 in. The structural framework for the lower 490 ft (see Fig. 1) consists of three columns with three planes of double X-bracing with transverse sheets and vertical hangers. The upper 124 ft consists of stiffened steel plates, which also form the outer surface of the Trylon. The columns for the lower

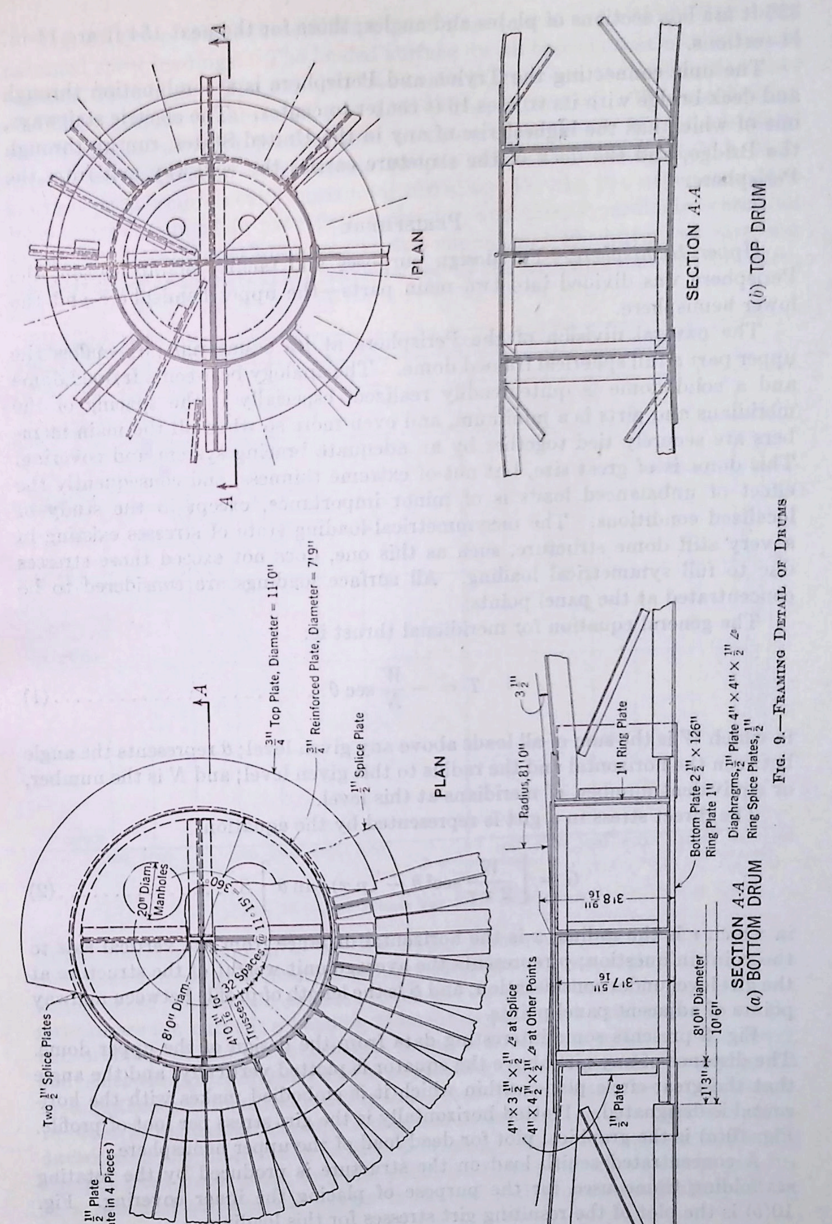


FIG. 9.—FRAMING DETAIL OF DRUMS

336 ft are box sections of plates and angles; those for the next 154 ft are 14-in. H-sections.

The unit connecting the Trylon and Perisphere is a combination through and deck bridge with its trusses 16 ft center to center. The electric stairways, one of which has the highest rise of any in the United States, run up through the Bridge, and the deck of the structure carries the walkway exit from the Perisphere.

Perisphere

Upper Hemisphere.—For design purposes and facility of fabrication, the Perisphere was divided into two main parts—the upper hemisphere and the lower hemisphere.

The natural division of the Perisphere at its equatorial girt makes the upper part a full spherical framed dome. The analogy between a framed dome and a solid dome is quite readily realized, especially if the spacing of the meridians and girts is a minimum, and even more so when all the main members are securely tied together by an adequate bracing system and covering. This dome is of great size, but not of extreme thinness, and consequently the effect of unbalanced loads is of minor importance, except in the study of localized conditions. The unsymmetrical-loading state of stresses existing in a very stiff dome structure, such as this one, does not exceed those stresses due to full symmetrical loading. All surface loadings are considered to be concentrated at the panel points.

The general equation for meridional thrust is:

$$T = -\frac{W}{N} \sec \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which W is the sum of all loads above any given level; θ represents the angle between the horizontal and the radius to this given level; and N is the number, or equivalent number, of meridians at this level.

The direct stress in a girt is represented by the equation:

$$G = \left[\frac{W}{2\,\pi\, r} \sec^2 \theta - (p\, x) \tan \theta \right] S \ldots\ldots\ldots\ldots\ldots (2)$$

in which r is the radius; x is the horizontal distance from the vertical axis to the point in question; p represents the average unit weight of the structure at the girt level under consideration; and S is the length of profile between midway points of adjacent panel points.

Fig. 8 presents some interesting data from the design of the upper dome. The distance of any girt above the equator is plotted vertically; and the angle that the great-circle plane within which it is embodied makes with the horizontal is designated. Plotted horizontally is the girt stress per foot of profile. Fig. 10(*a*) is the graphical plot for dead load of the upper hemisphere.

A concentrated zenith load on the structure is produced by the rotating scaffolding frame used for the purpose of placing the inner covering. Fig. 10(*b*) is the plot of the resulting girt stresses for this loading.

Fig. 10(*c*) represents three different sets of girt stresses for three differently assumed snow loadings. The loaded surface for all cases is that of a spherical segment 26 ft high. The uniform load curve is drawn on the assumption that the entire surface is subjected to a 25-lb-per-sq-ft loading. This assumption makes no allowance for the fact that, as the surfaces recede from the vertical axis of the Perisphere, they become steeper and hence cannot retain a 25-lb snow as do the more nearly horizontal surfaces. For the two other cases, the loading was assumed to have first a circular, and then a parabolic, variation from 25 lb per sq ft maximum to zero as a minimum. The latter two cases are perhaps closer to the actual load experienced by the structure than the first case.

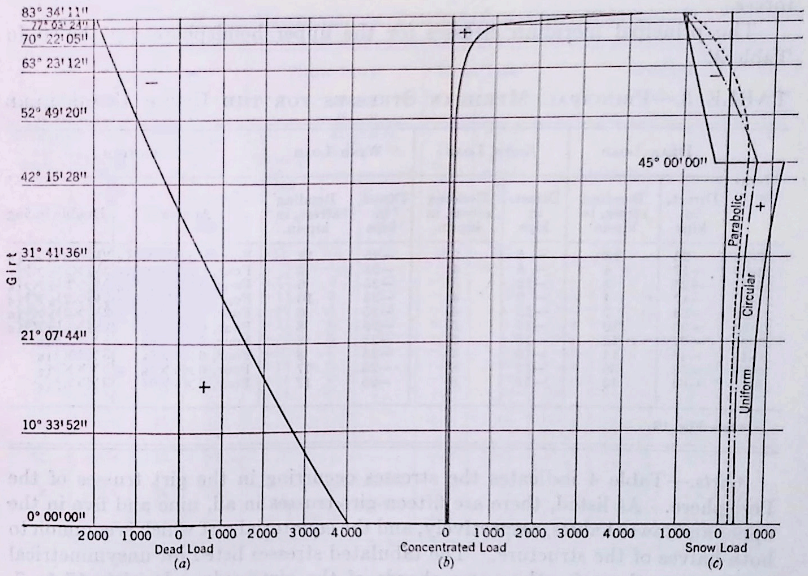


Fig. 10.—Girt Stress per Foot of Profile

All meridians and girts for the upper hemisphere, except the diagonal meridians and the girts near the top, are comprised of two-angle chord sections double laced with single angles. Since the chords of both the meridian and girt trusses are curved, a bending moment equal to the axial stress times the mid-ordinate of the curve between lacing intersections must be considered; and, since the girt chords act also as supporting members for the vertical purlin system, the unsymmetrical bending that occurs produces another set of stresses. In order to reduce the amount of unsymmetrical bending, a hanger was introduced from the intersection of the X-bracing. Since the bracing angles are straight from panel point to panel point, and the outer chord of the girt lies in a spherical surface, the hanger is skewed and produces a kick upon the trussed girt chord.

The center drum of the upper half of the Perisphere (see Fig. 9) consists of a flat $\frac{1}{2}$-in. plate 9 ft 3 in. in diameter acting as a splicing medium for the top of the eight meridians that attach to the drum as well as a top plate of the drum itself. The bottom plate of the drum is $\frac{1}{2}$ in. thick and 5 ft 8 in. in diameter. A $\frac{1}{2}$-in. ring splice plate is used for fixing the bottom chords of the eight meridians to the drum. The vertical ring of the drum is comprised of a $\frac{3}{4}$-in. plate 4 ft 9 in. deep and two angles 4 in. by 4 in. by $\frac{1}{2}$ in. bent to conform to a diameter of 5 ft 10 in. The meridian trusses attach to this vertical ring plate by means of 8-in. by 4-in. by $\frac{1}{2}$-in. angles. This detail, further aided by the two vertical cross diaphragms, made up of $\frac{1}{2}$-in. plates and 4-in. by 4-in. by $\frac{1}{2}$-in. angles, forms a rigid unit capable of transmitting the shearing forces.

The principal meridian stresses for the upper hemisphere are shown in Table 3.

TABLE 3.—PRINCIPAL MERIDIAN STRESSES FOR THE UPPER HEMISPHERE

Member[a]	Dead Load		Snow Load		Wind Load		Section	
	Direct, in kips	Bending stress, in kip-in.	Direct, in kips	Bending stress, in kip-in.	Direct, in kips	Bending stress, in kip-in.	Angles	Double lacing
27-26	−25	13	− 5	2	−25	12	Four, $6\times3\frac{1}{2}\times\frac{5}{16}$	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$
26-25	−17	4	− 8	4	−25	7	Four, $6\times3\frac{1}{2}\times\frac{5}{16}$	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$
25-24	−25	6	−12	3	−25	6	Four, $6\times3\frac{1}{2}\times\frac{5}{16}$	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$
24-23	−17	4	− 9	4	−25	10	Four, $6\times3\frac{1}{2}\times\frac{5}{16}$	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$
23-22	−24	7	−12	4	−25	6	Four, $6\times4\times\frac{3}{8}$	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$
22-21	−30	10	−16	5	−25	8	Four, $6\times4\times\frac{3}{8}$	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$
21-20	−37	12	−15	5	−25	8	Four, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
20-19	−46	14	−13	4	−25	8	Four, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
19-18	−53	36	−12	8	−25	17	Four, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
18-17	−64	43	−12	8	−25	17	Four, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$

[a] See Fig. 12.

Girts.—Table 4 indicates the stresses occurring in the girt trusses of the Perisphere. As listed, there are fifteen girt trusses in all, nine and five in the upper and lower halves, respectively, and the equatorial girt which is common to both halves of the structure. The tabulated stresses listed for unsymmetrical bending are those for the outer chords of the girts only. In girts 17 to 7, inclusive, the stresses resulting from snow load are included in the values listed under the dead-load columns. As will be noted, for main action, the girt truss at panel points 13 is completely idle and the one at panel point 7 partly so. In Fig. 8 is shown the one opening in the sphere, approximately 34 ft square, into which projects, without imposing any load on the Perisphere, the escalator and walkway unit—the Bridge. Being two panels in height, this aperture breaks the continuity of girt truss 13. As a result of this interruption, the lower half of the Perisphere was structurally designed as if girt 13 had been omitted; at the same time the girt was necessary at this location to carry the purlin systems, platforms, and coverings. The requirement that the structure be allowed to deform as designed, without any restraining action on the part of this discontinuous girt, was accomplished by making its panel-point connections with $\frac{7}{8}$-in. bolts in $1\frac{1}{16}$-in. holes, the nuts on the bolts being drawn up lightly and the threads checked. In adhering to the analysis of the lower hemisphere, and in order to control more closely the B_6 point, there must be no force exerted at the inner panel point of girt truss 7. Hence, these panel points were treated in the same manner as those of girt truss 13. The outer panel point of girt 7 has little or negligible dead-load stress, and as a result no control over the B_6 point; consequently it was riveted tight.

Girt 9 has an unsymmetrical inner flange whose section is comprised of an 8-in. by 6-in. angle and a 6-in. by 6-in. angle. The area provided by this inner flange is in excess of that required for stress, but was made necessary in order to accommodate the proper seating, connections, and framing for the bases of the moving platform columns.

TABLE 4.—STRESSES IN PERISPHERE GIRT TRUSSES

Girt member at panel point:	Dead Load			Snow Load			Wind Load		Section	
	Direct, in kips[a]	Bending Moments, in Kip[a]-In.		Direct, in kips[a]	Bending Moments, in Kip[a]-In.		Direct, in kips[a]	Bending moment, in kip[a]-in.	Angles	Double lacing
		Direct	Unsymmetrical		Direct	Unsymmetrical				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
26	− 20	9	28	−11	5	23	− 7	31	Two, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
25	− 19	10	28	−11	6	23	− 8	33	Two, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
24	− 17	6	14	−10	4	14	−11	17	Two, $6\times6\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
23	− 20	11	31	−11	6	26	−11	35	Two, $6\times6\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
22	− 17	6	11	− 9	3	10	−15	13	Four, $6\times3\frac{1}{2}\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
21	− 6	2	23	+20	7	22	−20	20	Four, $6\times3\frac{1}{2}\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
20	+ 9	4	37	+15	6	....	−20	28	Four, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
19	+ 26	17	37	+14	8	....	−22	24	Four, $6\times4\times\frac{3}{8}$	$3\times3\times\frac{5}{16}$
18	+ 44	32	30	+12	8	....	−25	34	Four, $6\times4\times\frac{7}{16}$	$3\times3\times\frac{5}{16}$
17	+164	46	30	....	....	....	−20	60	Four, $6\times6\times\frac{1}{2}$	$3\times3\times\frac{5}{16}$
15	+190	94	41	....	....	....	+42	80	Four, $6\times6\times\frac{9}{16}$	$3\frac{1}{2}\times3\times\frac{5}{16}$
13	....	....	26	....	....	....	....	....	Four, $6\times6\times\frac{1}{2}$	$3\frac{1}{2}\times3\times\frac{5}{16}$
11	+328	112	21	....	....	....	+35	+41	Two, $8\times8\times\frac{3}{4}$ Two, $8\times8\times\frac{7}{8}$	$4\times3\times\frac{5}{16}$
9	+109	34	16				− 8	32	One, $8\times6\times\frac{9}{16}$ One, $6\times6\times\frac{7}{16}$ Two, $6\times4\times\frac{9}{16}$	$4\times3\times\frac{5}{16}$
7	....	....	10				+21	42	Four, $6\times4\times\frac{7}{16}$	$4\times3\times\frac{5}{16}$

[a] 1 kip ("kilo-pound") = 1,000 lb

Lower Hemisphere.—The lower hemisphere of the Perisphere was designed by a process of trial and error, involving a series of converging adjustments in sectional areas, unit stresses, deformations, and total stresses. Each meridian truss below the equator was assumed to be a cantilever truss supported vertically on the ring girder and fixed horizontally at the vertical axis of the sphere; loaded at its upper end with the vertical loads from the upper hemisphere and at its various panel points with the vertical loads of the steelwork, inner and outer covering, circular platforms, equipment, and live loads; and restrained against outward movement by horizontal pulls from the various girts. The vertical loads were known, or could be assumed with sufficient accuracy. As a first approximation, a set of horizontal forces from the girts was assumed that was just large enough to balance the moment of the vertical forces. The stresses in the various members of the cantilever truss were then computed, sectional areas assumed, and the outward deflections of the various panel points of the truss computed by means of a Williot diagram. From the outward movement at the level of each girt, the resulting unit stresses in the girt could be computed. Since the first set of girt pulls had been assumed arbitrarily, the resultant horizontal displacements and girt stresses were not satisfactory, the cantilever trusses being bent considerably out of shape, and some of the girts showing very high tension and others considerable compression. After an examination of the results, a new set of horizontal pulls was assumed; the stresses, sections, deformations, and deflections of the cantilever truss recomputed; and the

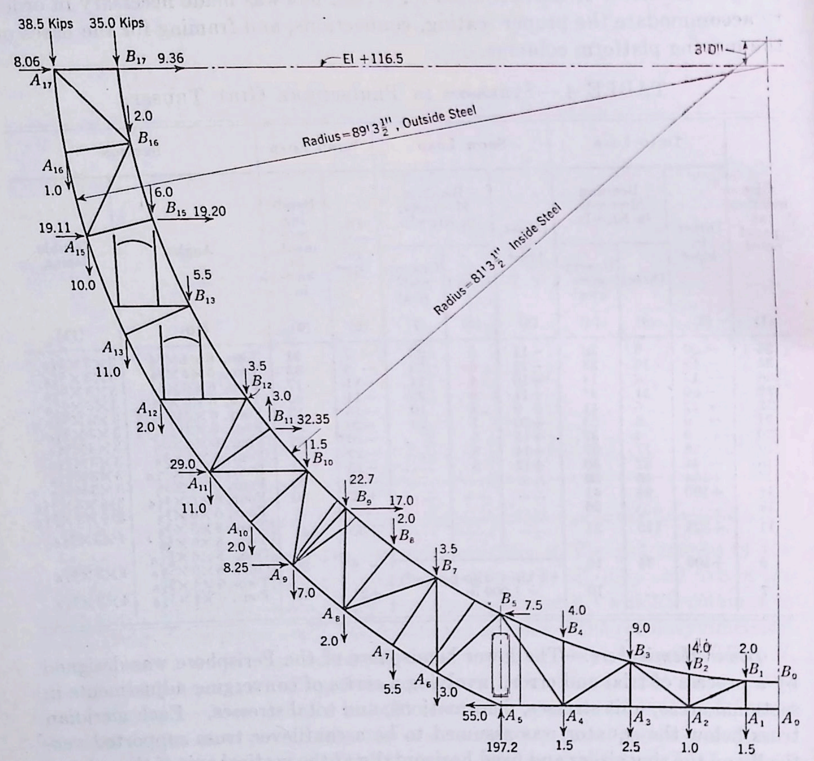


FIG. 11.—LOADING DIAGRAM; FINAL LOADS AND RING PULLS

resultant girt unit stresses again computed. After a number of successive trials, it was found possible to arrive at a series of horizontal pulls that would give reasonably uniform unit stresses of the desired intensity in a set of girts of moderate cross-sectional areas. It was found further that if any attempt were made to vary appreciably the various horizontal pulls thus determined, the cantilever truss would be forced considerably out of shape, and a balance between girt pulls, girt unit stresses, and girt sections could not be reached. In other words, it was found that the girt stresses could not differ appreciably

from those arrived at. A change of as little as 100 lb in the horizontal pull applied by a given girt would throw the cantilever truss appreciably out of shape. It was also found that the cantilever trusses would be very rigid, since the girts would permit only very small transverse movements.

The action of the steel framework of the Perisphere under the dead, live, and snow loads is one in which the meridians, in compression, tend to bow out; this tendency in turn brings the girts into play as tension rings. One can have a very clear mental picture of the entire structural action by pressing uniformly on top of a rubber ball set in a ring collar about one third the diameter of the ball. In reacting to these dead, live, and snow loads the Perisphere will deform from the outline of a true sphere. When it is subjected to final loads and ring pulls as indicated in Fig. 11, the Perisphere will deform so as to allow a point on the equator to move radially 0.25 in. and vertically 0.5 in. The principal meridian stresses for the lower hemispheres are shown in Table 5.

TABLE 5.—PRINCIPAL MERIDIAN STRESSES FOR THE LOWER HEMISPHERE

(a) OUTSIDE STEEL							(b) INSIDE STEEL			
			Section							
			Typical		Entrance					
Member[a]	Direct load,[b] in kips	Bending stress,[b] in kip-in.	Two angles	One plate	Two angles	One plate	Member[a]	Direct load,[b] in kips	Bending stress,[b] in kip-in.	Typical section (two angles)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)
A17-A15	−27	50	6×6×7/16	....	....	....	B17-B16	−38	63	6×6×7/16
A15-A11	−25	56	8×6×9/16	....	....		B16-B15	−59	100	6×6×9/16
A11-A9	−53	56	6×6×7/16	7×5/8	6×6×1/2	7×5/8	B15-B12	−103	257	8×6×9/16
A9 -A8	−65	86	6×6×7/16	7×5/8	6×6×1/2	7×5/8	B12-B10	−89	46	8×6×9/16
A8 -A7	−113	97	8×6×9/16	7×5/8	8×6×11/16	7×5/8	B10-B9	−122	114	8×6×3/4
A7 -A6	−117	97	8×6×9/16	7×3/4	8×6×11/16	7×3/4	B9 -B7	−127	145	8×6×3/4
A6 -A5	−209	....	8×8×13/16	....	....	....	B7 -B5	−51	93	8×8×13/16
A5 -A4	−132	....	8×8×3/4	6½×3/4	8×8×7/8[c]	....	B5 -B3	+40	62	8×8×3/4
A4 -A2	−101	....	8×8×3/4	....	....	....	B3 -B1	−5	....	8×8×3/4
A2 -A1	−70	....	8×8×3/4	....	....	....	....	....	....	....

[a] See Fig. 12. [b] Dead load plus live load. [c] At columns.

The emergency exit fire passageways contained between the outer and inner shells of the Perisphere at the 75.5-ft and 87.5-ft levels necessitated the plate section of meridian truss between the 12th and 15th panels.

In Table 5(a) (lower hemisphere) it will be noted that the sections of the outer meridian chords between panel points 11 and 6 are not the usual two-angle section, but have plates 7 in. deep placed between the angles and extending from gusset plate to gusset plate, but not developed into the joints. This type of member affords ample chord section for direct stress in the panel point region as well as for the increased stress due to bending at the mid-panel point. Table 5 (lower hemisphere) indicates heavier chord members below panel point 9 at the entrance side of the Perisphere, which was done to accommodate the increase in stresses due to the extra dead and live loads concentrated at the junction of the Perisphere and Bridge. The net unbalanced steel load of 50,000 lb at the entrance to the Perisphere is partly counteracted by placing, advantageously, some of the heavier electrical equipment in the four bays diametrically opposite the entrance. Another aid in balancing the extra load at the entrance is the placing of a concrete carrying slab for the electrical apparatus as compared with the lighter composition floor on the entrance platforms.

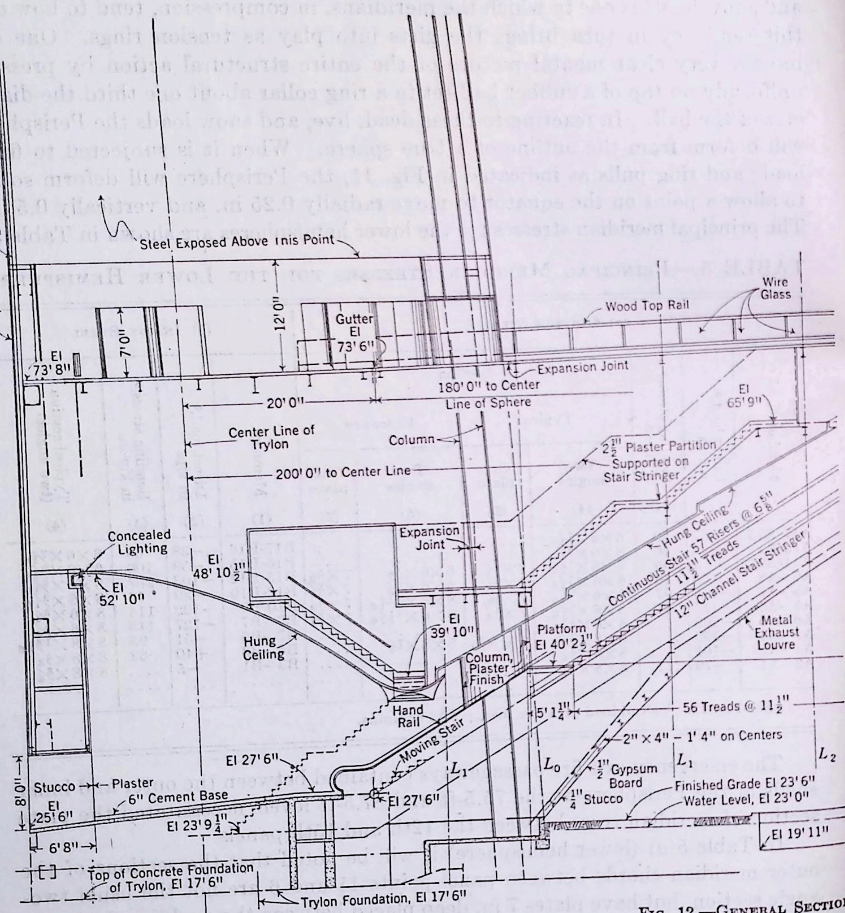
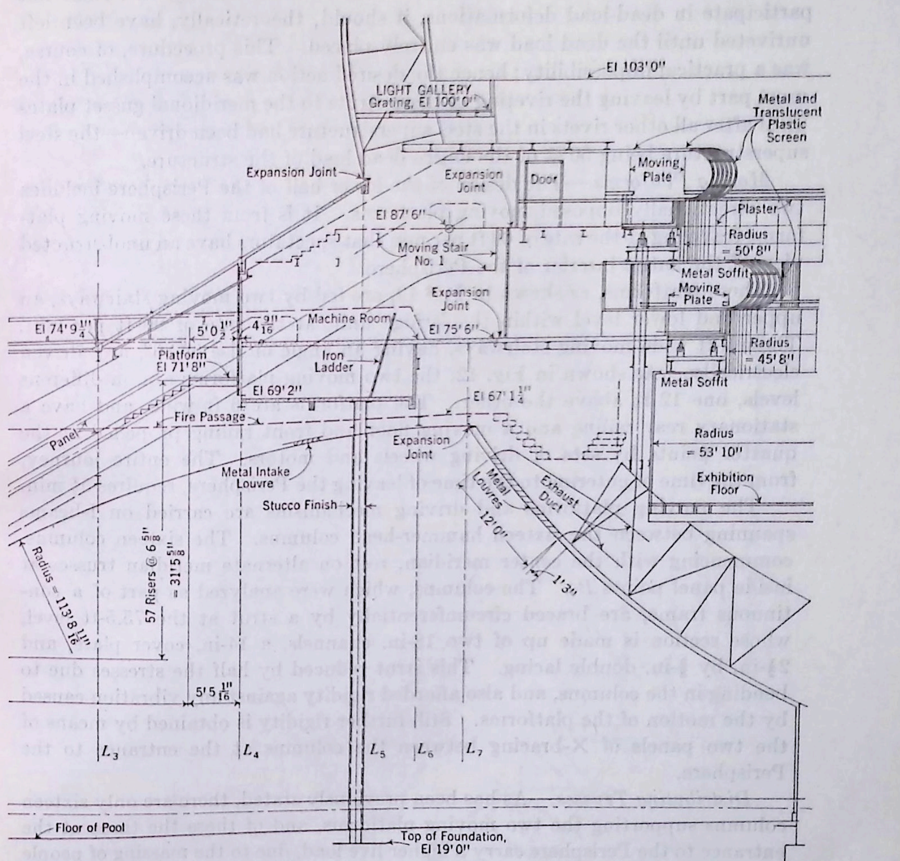


Fig. 12.—General Section

In the final analysis, due to the nature of the structure (that is, its geometrical arrangement), the Perisphere framework was found to be extremely sensitive to even small changes in the assumed distribution of stresses in the various parts. This supersensitive characteristic of the structure is a measure of design safety. Since the entire framework strives to distribute quickly all types of loadings, it is of prime importance that very careful study be accorded the connections and framing of all members. Inasmuch as the structure was

designed to conform to particular deformations, resulting in specified sets of stresses, care was taken to avoid, as far as it was physically possible, any interference by members or their connections with the desired deformations of the structure.

The ring girder as a member receives its stress from bending due to vertical



Through Theme Center

loads and ring shortening caused by meridional thrusts. The chords of the meridian trusses pass above and below the flanges of the ring girder. This arrangement facilitated fabrication and erection but complicated the connection for transferring meridional thrusts into the ring girder; and a further complication which makes this particular connection of the utmost importance is the abrupt change in the path of the outer chord stresses caused by the outer meridional chords turning horizontally at this point.

In the outer cross bracing system, just outside the ring girder, there is a two-angle wind strut connected to the bottom cover plates of the ring girder by $\frac{5}{8}$-in. bent plates. By means of this device the wind shear is transferred into the bottom girder flange. In order that this strut, which is in reality an auxiliary ring flange, shall be reserved for wind action only and shall not participate in dead-load deformations, it should, theoretically, have been left unriveted until the dead load was entirely placed. This procedure, of course, was a practical impossibility; hence the desired action was accomplished in the most part by leaving the riveting of these struts to the meridional gusset plates until after all other rivets in the steel superstructure had been driven—the steel superstructure being 50% of the entire dead load of the structure.

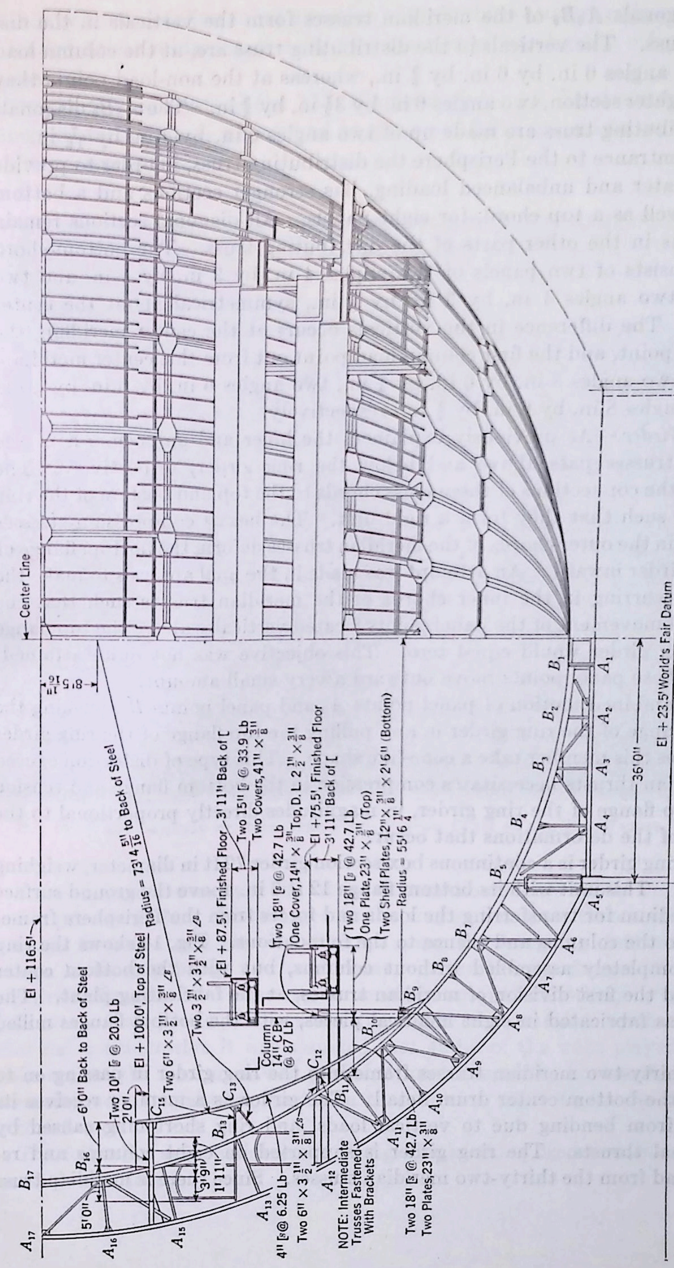
Moving Platforms.—The design of the lower half of the Perisphere includes two directionally opposed moving platforms. It is from these moving platforms traveling at the rate of 60 ft per min that spectators have an unobstructed view of the entire interior of the Perisphere.

These platforms, as shown in Fig. 12, are fed by two moving stairways, an upper and lower level within the Bridge unit, at the rate of 90 ft per min. These 2-ft wide moving stairways, having an angle of rise of 30°, are driven electrically. As shown in Fig. 12, the two moving platforms are on different levels, one 12 ft above the other. The platforms are 6 ft wide, and have a stationary rear railing and a moving floor and front railing propelled at the quarter points by sets of driving wheels and motors. The entire journey, from the time of entering to the time of leaving the Perisphere, requires $5\frac{1}{2}$ min.

The moving platforms and driving mechanisms are carried on I-beams spanning between the sixteen hammer-head columns. The sixteen columns, commencing with the center meridian, rest on alternate meridian trusses at inside panel points B_9. The columns, which were analyzed as part of a continuous frame, are braced circumferentially by a strut at the 75.5-ft level, whose section is made up of two 18-in. channels, a 14-in. cover plate, and $2\frac{1}{2}$-in. by $\frac{3}{8}$-in. double lacing. This strut reduced by half the stresses due to bending in the columns, and also afforded rigidity against any vibration caused by the motion of the platforms. Still further rigidity is obtained by means of the two panels of X-bracing between the columns at the entrance to the Perisphere.

Distributing Trusses.—As has been previously stated, there are only sixteen columns supporting the two moving platforms, and of these the three at the entrance to the Perisphere carry a higher live load, due to the massing of people in this vicinity, and a heavier dead load because of the extensive framing and partitioning necessary in order to provide the required floor areas (see Fig. 13).

These sixteen columns find their seats on sixteen meridian trusses at the inside panel points B_9. Since there are thirty-two meridians that must be made to share alike in carrying this platform loading to produce uniform deformation of the framework, a truss was designed to frame between meridians at the load-application points. The inside chord of the girt at panel points B_9 serves also as the top chord for the distributing truss.



QUARTER SECTION

FIG. 13.—STEEL FRAMING, LOWER HEMISPHERE

The diagonals A_8B_9 of the meridian trusses form the verticals in the distributing truss. The verticals in the distributing truss are, at the column load points, two angles 6 in. by 6 in. by $\frac{3}{8}$ in., whereas at the non-load points they are of the lighter section, two angles 6 in. by $3\frac{1}{2}$ in. by $\frac{3}{8}$ in. The A_8B_9 diagonals of the distributing truss are made up of two angles 6 in. by 4 in. by $\frac{7}{16}$ in.

At the entrance to the Perisphere the distributing truss, in order to provide for the greater and unbalanced loading, has stronger sections and a bottom chord, as well as a top chord, for eight panels. All diagonal sections remain the same as in the other parts of the distributing truss. The bottom-chord section consists of two panels of two angles 4 in. by 3 in. by $\frac{1}{2}$ in. and two panels of two angles 4 in. by 3 in. by $\frac{3}{8}$ in., symmetrical about the center meridian. The difference in the verticals occurs at the center meridian, the first panel point, and the first column load point out from this center meridian; these are two angles 8 in. by 6 in. by $\frac{3}{4}$ in., two angles 6 in. by 6 in. by $\frac{1}{2}$ in., and two angles 8 in. by 6 in. by $\frac{1}{2}$ in., respectively.

Ring Girder.—As previously explained, the inner and outer chords of the meridian trusses pass above and below the ring girder, respectively. The details of the connections of these truss chords to the top and bottom of the ring girder are such that they form a rigid unit. The heavy compression stresses occurring in the outer chords of the meridian trusses deform the bottom flange of the ring girder inward. An attempt was made in the final analysis to make the stresses occurring in the inner chords of the meridian trusses such that the horizontal movement of the panel points located vertically above the top flange of the ring girder would equal zero. This objective was not quite attained, because these panel points move outward a very small amount.

The combined motion of panel points A_5 and panel points B_5, pushing the bottom flange of the ring girder in and pulling the top flange of the ring girder out, makes this member take a cone-like shape. This type of distortion caused by meridian thrusts necessitates compression in the bottom flange and tension in the top flange of the ring girder, in magnitudes directly proportional to the amount of the deformations that occur.

The ring girder is a continuous box-section girder 72 ft in diameter, weighing 139 tons. This unit with its bottom flange 12 ft 6 in. above the ground surface is the medium for transferring the loads and forces from the Perisphere framework into the columns and thence to the foundations. Fig. 14 shows the ring girder completely assembled without columns, but with the bottom center drum and the first division of meridian trusses, at the fabricating plant. The girder was fabricated in eight identical pieces, with the bottom flanges milled to bear.

All thirty-two meridian trusses frame into the ring girder in passing on to butt at the bottom center drum detail. The girder as a member receives its stresses from bending due to vertical loads, and ring shortening caused by meridional thrusts. The ring girder is supported on eight columns and receives load from the thirty-two meridian trusses. Since there is meridian truss at each column, it follows that a section of girder between two adjacent columns is loaded at its midpoint and its two quarter points by the other meridian trusses.

In the final analysis of the 72-ft diameter girder for direct stress as a ring, panel points B_5 and A_5 were found to move out 0.015 in. and to move in 0.058 in., respectively.

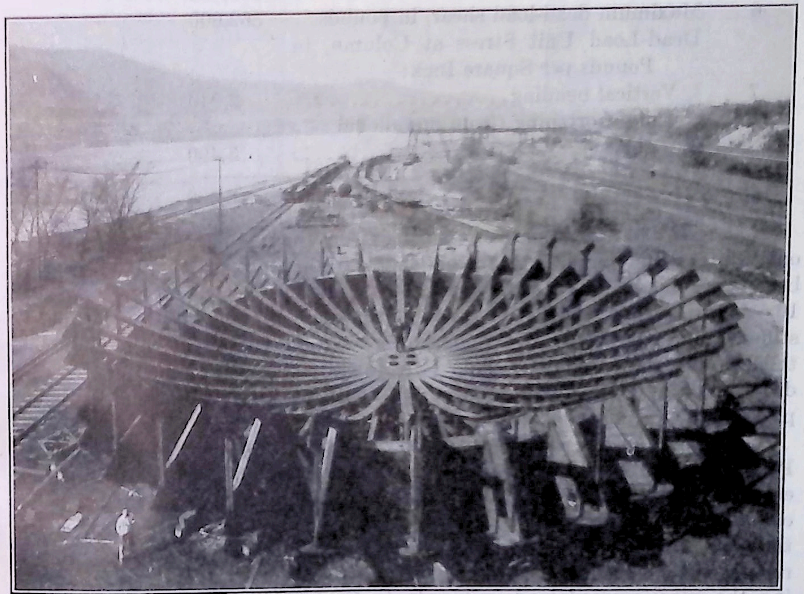


FIG. 14.—TEST ASSEMBLY OF RING GIRDER AND BOTTOM DRUM AT THE FABRICATING PLANT

The assumption was made that these deformations would create ring stresses in the circular girder. The flanges of the ring girder were then sectioned so as to produce the unit stresses accompanying the foregoing deformations—tension in the top flange and compression in the bottom flange. Since the ring-girder flanges are necessarily connected, one to the other, by the two $\frac{7}{16}$-in. webs, some participation in the ring action must be taken by these webs. After due consideration it was assumed that 45 in. of the webs played a part in the action, the maximum web participation being at the bottom flange and decreasing as a straight-line variation to zero at a point 45 in. up from the bottom flange.

The following data indicate the ring-girder sections used and the stresses from balanced loads:

Item	Description	
	Section:	
1	Two web plates	90 in. by $\frac{7}{16}$ in.
2	Four angles	8 in. by 6 in. by $\frac{3}{4}$ in.
3	One top cover plate	23 in. by $\frac{5}{8}$ in.
4	Two bottom cover plates	23 in. by $1\frac{3}{16}$ in.
5	Maximum dead-load moment, in pound-feet	2,133,000
6	Maximum dead-load shear, in pounds	362,000
	Dead-Load Unit Stress at Column, in Pounds per Square Inch:	
7	Vertical bending	2,940
8	Ring shortening (from meridional thrust)	3,490
9	Total	6,430

Bottom Drum.—After passing the ring girder the meridian trusses converge on a center drum detail. All bottom chords of the thirty-two meridian trusses bear with milled surfaces against the bottom plate of the center drum which is 10.5 ft in diameter and $2\frac{1}{2}$ in. thick, as shown in Fig. 9. A 1-in. ring plate stiffened by plate and angle diaphragms, a top plate 11 ft in diameter and $\frac{3}{4}$ in. thick, and a top reinforcing plate $\frac{3}{4}$ in. thick make up the remainder of the drum. All meridian trusses are rigidly framed into the top, bottom, and ring plates of the drum detail.

Air Conditioning.—The complete air conditioning system provided for the Perisphere requires a fully equipped fan and compressor room. This room and equipment were placed within the ring girder and supported on the inner flanges of the meridian trusses. The reinforced concrete platform forming the floor of the room is placed on a series of circular, concentric, reinforced concrete beams resting on the panel points of the meridian flanges and in this manner distributing the superimposed load uniformly to the meridians. This load, placed within the ring girder, actually decreases somewhat the meridian stresses resulting from the action of the framework. The accompanying ductwork and details of the system are contained within the space provided between the outer and inner shells, and the necessary louvers and vents are provided near the top of the Perisphere.

Columns.—The make-up of the eight main columns, each weighing 9 tons, supporting the Perisphere is as shown in Fig. 15. Because of the framing of these columns into the ring girder and meridian trusses, they are fixed at the top. At the bases the columns are fixed, by anchor bolts 3 in. in diameter, about the radial axis 2–2; but in order to reduce temperature stresses the columns are hinged about the tangential axis, 1–1.

The hinging effect was accomplished, as will be noted in Fig. 15, by tapering the 5-in. base plate and providing the 8-in. by 8-in. by $\frac{1}{2}$-in. breathing angles.

Wind Shear on Columns (See Fig. 16).—The distribution of the wind shear among the various columns was determined in the following manner: All columns are alike. At the bottom they are pin-ended about axis 1–1 and fixed-

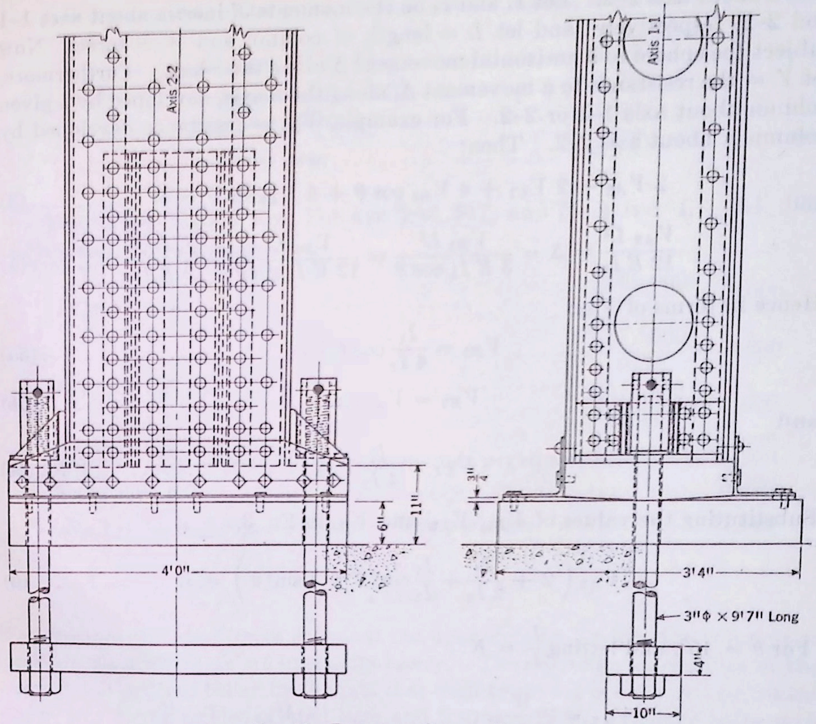


FIG. 15.—DETAIL OF COLUMN BASE

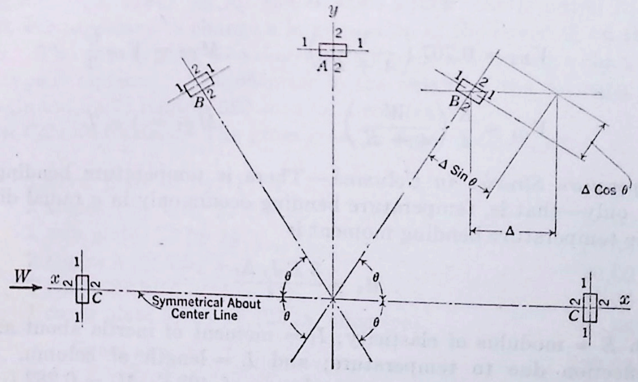


FIG. 16

ended about axis 2–2. Let I_1 and I_2 be the moments of inertia about axes 1–1 and 2–2, respectively; and let L = length of column and W = force. Now subject the sphere to a horizontal movement Δ along the x-axis. Furthermore, let V = the resistance to a movement Δ, along the x-axis, developed by a given column about axis 1–1 or 2–2. For example, V_{A2} = resistance developed by column A about axis 2–2. Then:

$$2\,V_{A2} + 2\,V_{C1} + 4\,V_{B1}\cos\theta + 4\,V_{B2}\sin\theta = W \qquad (3)$$

and

$$\frac{V_{A2}\,L^3}{12\,E\,I_2} = \Delta = \frac{V_{B1}\,L^3}{3\,E\,I_1\cos\theta} = \frac{V_{B2}\,L^3}{12\,E\,I_2\sin\theta} = \frac{V_{C1}\,L^3}{3\,E\,I_1} \qquad (4)$$

Hence in terms of V_{A2}:

$$V_{B1} = \frac{I_1}{4\,I_2}\cos\theta\,V_{A2} \qquad (5a)$$

$$V_{B2} = V_{A2}\sin\theta \qquad (5b)$$

and

$$V_{C1} = \frac{I_1}{4\,I_2}\,V_{A2} \qquad (5c)$$

Substituting the values of V_{B1}, V_{B2}, and V_{C1} in Eq. 3,

$$V_{A2}\left(2 + \frac{I_1}{2\,I_2} + \frac{I_1}{I_2}\cos^2\theta + 4\sin^2\theta\right) = W \qquad (6)$$

For $\theta = 45°$, and letting $\frac{I_1}{I_2} = K$:

$$V_{A2} = \frac{W}{4+K}; \qquad M_{A2} = V_{A2}\frac{L}{2}$$

$$V_{B1} = \frac{0.707}{4}\,K\left(\frac{W}{4+K}\right); \qquad M_{B1} = V_{B1}\,L$$

$$V_{B2} = 0.707\left(\frac{W}{4+K}\right); \qquad M_{B2} = V_{B2}\frac{L}{2}$$

and

$$V_{C1} = \frac{K}{4}\left(\frac{W}{4+K}\right); \qquad M_{C1} = V_{C1}\,L$$

Temperature Stresses in Columns.—There is temperature bending about axis 1–1 only—that is, temperature bending occurs only in a radial direction. Then the temperature bending moment is

$$M_t = \frac{3\,E\,I_1\,\Delta_t}{L^2} \qquad (7)$$

in which E = modulus of elasticity; I_1 = moment of inertia about axis 1–1; Δ_t = deflection due to temperature; and L = length of column. Substituting the values for a temperature change of 40° F: $M_t = 0.282\,I_1$ kip-in. $= \frac{0.282 \times 13{,}561}{12} = 319$ kip-ft.

Column Section.—The gross area, in square inches, is:

4 plates 30 by $\frac{11}{16}$	82.52
4 angles 8 by 4 by $\frac{3}{4}$	33.76
2 plates 20 by $\frac{5}{8}$	12.50
Gross area	128.78

The section moduli are: For axis 2–2, 917; and for axis 1–1, 1,164. The unit stresses are as follows:

Direct stress	$\frac{1{,}022{,}000}{128.78} = 8{,}000$
Bending stress	$\frac{7{,}872{,}000}{917} = 8{,}600$
	$\frac{1{,}915{,}000}{1{,}164} = 1{,}600$
Dead + Live + Snow + Wind, in pounds per square inch	18,200
Temperature	3,300
Dead + Live + Snow + Wind + Temperature	21,500

Trylon

Because the wind forces governed the design of the Trylon, the columns as well as the anchorages are unusually heavy. The wisp-like appearance of the completed tower is belied by the fact that each tower-leg is anchored by means of a very heavy welded-riveted base and fourteen $2\frac{7}{8}$-in. diameter bolts each 12 ft in length. The base and the anchor bolts of each column are embedded in a three-way reinforced mass of concrete. Special precautions were taken in designing each tower leg for the 855-ton uplift. As is noted in the data table, it was necessary to change a large portion of the covering on the Trylon in 1940. The new type of covering weighed only a little more than one third of the type it replaced. The decrease in the weight of the covering increased the maximum uplift load to 920 tons on a column.

First Column Section.—The gross area, in square inches, is:

1 cover plate 22 by $1\frac{1}{8}$	24.75
2 angles 8 by 6 by 1	26.00
2 web plates 32 by $1\frac{1}{2}$	96.00
2 angles 8 by 8 by 1	30.00
2 plates 24 by 1	48.00
1 cover plate 37 by $1\frac{1}{8}$ (23 in. effective)	25.88
Gross area	250.63

$\frac{l}{r} = 32$; allowable f_c = 14,740 lb per sq in.

The unit stresses, in pounds per square inch, are as follows:

$$\text{Dead load} \ldots\ldots\ldots \frac{1,100,000}{250.63} = 4,400$$

$$\text{Wind load} \ldots\ldots\ldots \frac{2,580,000}{250.63} = 10,300$$

$$\text{Dead} + \text{wind} \ldots\ldots\ldots\ldots\ldots\ldots\ 14,700$$

Coverings

The exterior covering of the Perisphere consists of 2-in. by 4-in. wood nailers, treated with a fire-retardant medium and screw-fastened to the vertical steel purlin system; two thicknesses of ½-in. gypsum wallboard, the first layer

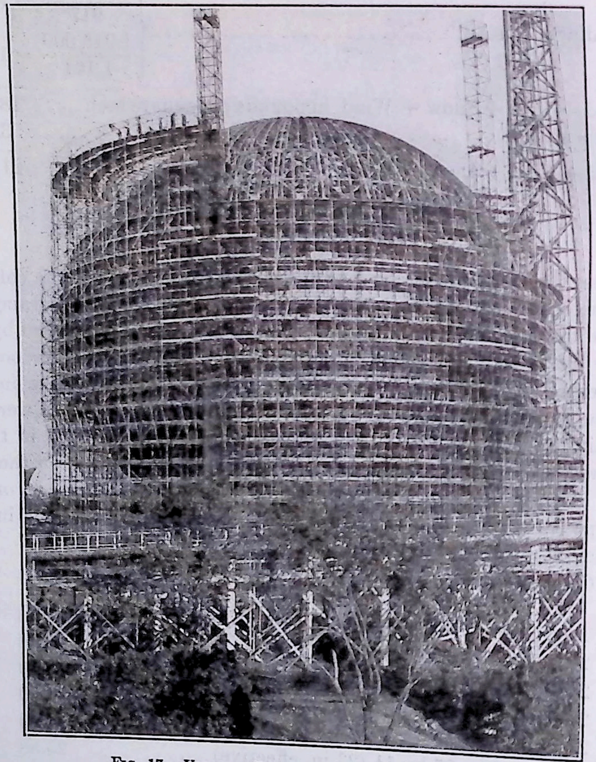


Fig. 17.—View of Demountable Scaffold System

being damp-proofed with a full covering of asphalt emulsion, were applied vertically and nailed to the horizontal timber furring. Then three layers of a magnesite compound, reinforced with two layers of jute fabric, were applied to the entire surface of the Perisphere and painted white.

The interior shell or lining of the Perisphere consists of an acoustical tile, upon which pictures are projected. The tile is nailed to 2-in. by 3-in. fire-proofed horizontal furring, 27 in. on centers, attached to the inner steel purlins by means of heavy wire clips. On the area of tile extending from the offset in the inner shell below the equator, to a point approximately 30° from the zenith, the tile has a backing in alternate panels, in both directions, of a ⅜-in. plasterboard and a 1-in. sound-absorbing board.

The exterior covering applied to the steel purlins of the Trylon and Bridge was of the same type as that for the Perisphere except that there was only one thickness of gypsum wallboard. The outer coverings of the Perisphere, Trylon, and Bridge were applied by using an exterior, steel-pipe, demountable scaffolding system, as shown in Fig. 17.

The interior shell of the Perisphere above the level of the light-projector platform was constructed with the use of a revolving scaffold made of tied-arch trusses having their curvature fitted to the inner Perisphere surface. The three fan-like trusses were interlocked with X-bracing. The scaffold was suspended from the zenith of the sphere by a spider-and-pivot hanging support. At the light gallery (100-ft level), the scaffold has a set of rollers, one under each truss, and these rollers run on a circular I-beam track. By means of horizontal platforms projecting out at various levels from the arched scaffold, workmen were able to apply a portion of a lune of inner surfacing and then move the scaffold around on its track into a new position, lock the rollers, and apply the surfacing to another lune. This procedure was repeated until the upper inner shell was completed.

The tied-arch scaffold framework, being bolted together, was easily dismantled upon completion of the surface, and, if necessary, can be put up again readily for maintenance purposes.

Fabrication

Both the 1,000 tons of Trylon steel and the 2,000 tons of Perisphere steel were fabricated and erected by the American Bridge Company.

It was necessary to curve all members of the Perisphere, except the columns, webs of trusses, and interior framing, in order to fit the spherical surfaces. As a result of bending these members to arcs of great circles, as previously mentioned, the amount of work was held down to a minimum. More than one quarter of a million rivets, of which 100,000 were used during erection, were driven in order to fabricate and erect the 6,600 individual pieces of the Perisphere, varying in weight from a few pounds to 18 tons.

The curved members were bent cold by the use of jigs and bending machines. All curved trusses of the Perisphere were laid out and fabricated in specially designed jigs. The field connections in the meridian trusses were reamed assembled so as to minimize the inaccuracies and assure the best obtainable construction.

The bottom center drum and the ring girder, constituting the "nucleus" from which the Perisphere was constructed, are shown assembled at the fabricator's plant in Fig. 14. The ring girder was fabricated in eight sections and fitted to form a perfect circle 72 ft in diameter.

Erection

Trylon.—The 615-ft Trylon was erected by means of a 70-ft basket boom. The column steel came in sections varying from 19 ft to 33 ft in length, and when the basket boom had surrounded itself with tower framework, it was lifted and the guys fastened in higher positions provided for in the column details. By this method the Trylon, including the top steel-plate section, was completely erected.

Perisphere.—A caterpillar-tractor crane having a 100-ft boom and a 12-ft jib was used to erect the Perisphere steel to within a short distance of the equator. Before reaching this stage of construction, however, the tractor crane set up an erection tower 30 ft square and 90 ft high on the inside of the Perisphere. The tower rested on, and was fastened to, the inner chords of the meridian trusses within the ring girder. Two 12-ton stiff-leg derricks set on diagonally opposite corners of the tower erected the remainder of the Perisphere with their 97-ft booms.

In general, the procedure of erecting the Perisphere was that of first placing one section of each of the meridians and then inserting the corresponding units of the girt trusses. Then the next sections of meridian and girt trusses were bolted up, and so the sequence was repeated until completion. The riveting followed the erection very closely so as to have all connections riveted before very much load came on the framework. No camber was provided in any part of the structure; consequently, the completed Perisphere has its natural dead-load deformation.

Due to the process of erection (placing a complete section of corresponding meridians and girts before proceeding to the next section) all members were erected in practically an unstressed condition. At some stages of the erection, the unsymmetrical temperature effects on the steelwork were enough to create gaps of $1\frac{1}{2}$ in. on closing units of girt trusses. This opening was taken care of by adjustment back into all of the thirty-two units of a girt truss so that in following such method no reaming of any connections was necessary in the entire erection of all the meridians and girts.

It is worthy of note that, although these structures were unusual in both type and details, no unforeseen difficulties arose, during either the fabrication or the erection, that involved delays or required revisions.

Acknowledgments

The Theme Center was built by the New York World's Fair 1939, Inc., under the direction of the Department of Construction, with John P. Hogan, President, Am. Soc. C. E., chief engineer, and L. B. Roberts, M. Am. Soc. C. E., assistant chief engineer.

The design and plans of the steelwork for the Perisphere, Trylon, and Bridge were prepared by Waddell and Hardesty under the general direction of Mr. Hardesty, those for the Perisphere being supervised by Mr. Hedefine, and those for the Trylon and Bridge by Francis De Schauensee, M. Am. Soc. C. E. The foundations were designed by Moran, Proctor and Freeman, and the architects were Harrison and Fouilhoux. The original manuscript, complete with bibliography, has been placed on file for reference at The Engineering Societies Library, 29 West 39th Street, New York, N. Y.

DISCUSSION

Louis Balog,[5] Esq. (by letter).—The five Perisphere designs mentioned by the authors were submitted to the Construction Department of the Fair for consideration on April 12, 1937. One of these, designated as Design No. 1, was adopted for construction. The major part of the paper presents a detailed account of this design.

Design No. 1 consists of a steel framework and a separate outer covering. The spherical steel framework is composed of girt and meridian trusses of a depth of $\frac{1}{16}$ to $\frac{1}{8}$ of the outside radius. Assuming pin connections at the intersection of the meridians and girts the stresses in the framework, due to symmetrical loads, can be determined from equilibrium requirements. If the connections are rigid, the elastic deformation of the girts results in meridian moments. The magnitude of these moments is primarily a function of the moment of inertia of the meridians. The deeper the meridians the larger the moments will be due to elastic deformation of the girts. In case of n girts the meridian moments can be obtained by the solution of $n - 1$ elastic equations expressing the equality of the elastic displacements of girts and meridians at their intersections. The operations involved are not too formidable; but they still are not justified by the significance of these moments as compared to those that result from the curvature of the meridians between two intersection points. For unsymmetrical loadings, like those due to wind, the number of elastic equations increases so far beyond any reasonable limit that a rigorous investigation becomes impracticable.

To simplify the analysis the authors divide the sphere at the equator. They assume the upper hemisphere to be a statically determinate structure and obtain the stresses from equilibrium requirements. In the lower hemisphere the meridians were assumed to be cantilevers resting on a series of elastic supports, represented by the girts. These assumptions result in a sufficiently close approximation of the stresses in the framework due to axially symmetrical loadings, such as the dead and snow loads. The wind-pressure distribution, as given from tests, is not symmetrical to any plane containing the vertical axis of the sphere. It is not adaptable to mathematical expression that is suitable for the derivation of closed formulas for the wind stresses. The authors' analysis resulted in safe values for the stresses produced by the assumed wind-pressure distribution as comparative calculations made by the writer show. The diagonal bracing provided, and the large moment-carrying capacity of the deep meridian trusses, make this structure well fitted to carry unsymmetrical wind loads.

Although rigorous investigations indicated that the approximate methods used by the authors are suitable for design purposes, it is regrettable that no stress measurements were made on the structure as built. Such measurements would have given invaluable hints for correct simplified analysis, the only kind that is of genuine value in the design of this type of complex structure. This holds especially for the wind stresses. Extensive laboratory investiga-

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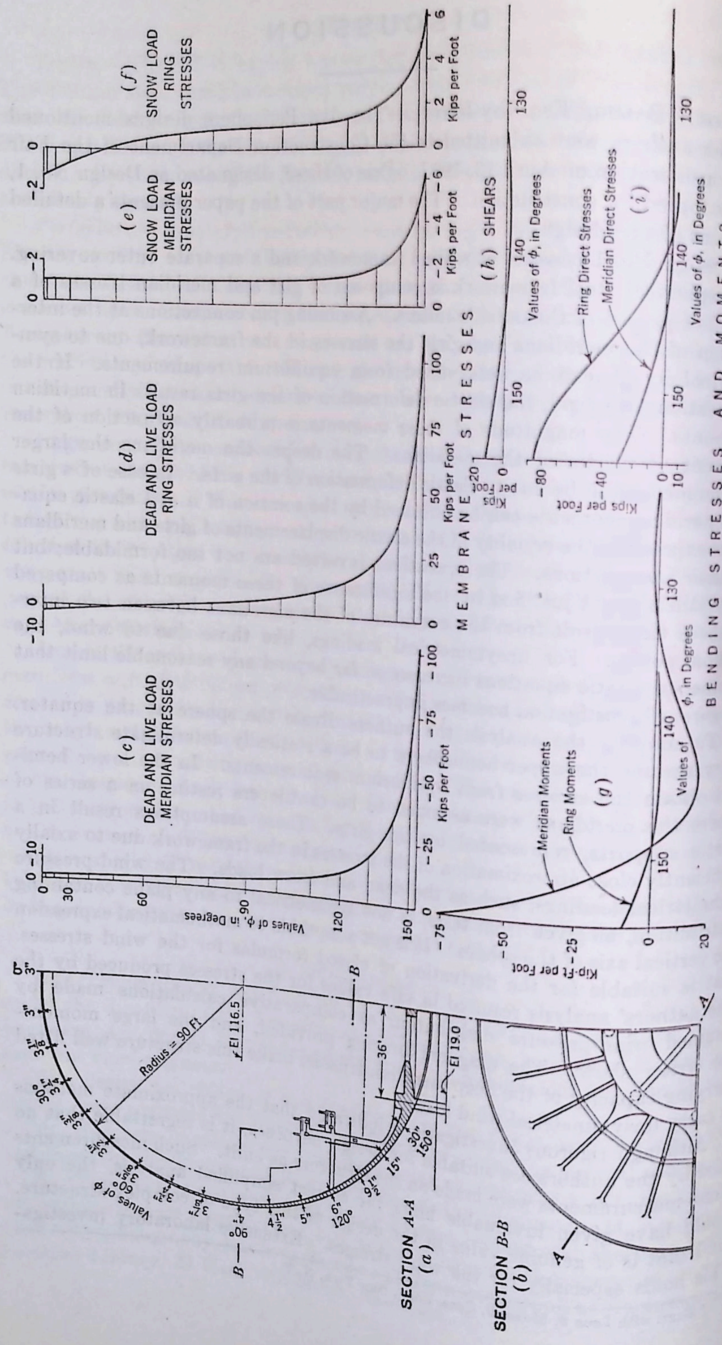

Fig. 18.—Sections and Principal Stresses of Concrete Perisphere

tions were made to determine the magnitude and distribution of the wind loads. The stresses produced by these loads, however, were not ascertained by an experimental method. The authors' skilful estimate of the stresses in the framework due to wind would be of considerably greater value if measurements had shown how closely the actual conditions were approximated.

The problem in the design of the Perisphere was to create a structure which carries specified loads safely and has a smooth spherical surface. The latter requirement necessitates the use of a complicated outside covering on a steel framework. By the use of reinforced concrete, such covering is eliminated and the carrying structure can be formed readily to the desired shape and surface.

Design No. 5, a reinforced concrete shell structure, was proposed and designed by the writer. It differs, not only in its material but also in structural conception, from the other four designs, which were ribbed steel structures. The principal characteristics, design data, and quantities of this design are presented in the following:

The great carrying capacity of shells in general and that of shells of double curvature, like the sphere, in particular, would permit the use of very thin sections. In the upper two thirds of this spherical shell the thickness was determined by construction considerations rather than by the stresses, the safety factor against buckling, or durability requirements. The construction of a sufficiently precise formwork and the assurance of careful workmanship in placing the reinforcing steel and concrete constituted the difficulties that influenced the selection of the minimum thickness. These considerations resulted in the adoption of greater thickness for the upper part of the shell than that of existing permanent structures with similar radius of curvature.

Fig. 18 indicates the principal dimensions and stresses of the reinforced concrete sphere. The outside diameter is 180 ft. It is supported on eight round columns, 4 ft in diameter, arranged similarly to the steel design.

The thickness of the shell increases slowly from 3 in. $\left(\frac{1}{360}\right.$ of the outside radius$\left.\right)$ at the top where $\phi = 0°$, to 4 in. $\left(\frac{1}{270}\right.$ of the outside radius$\left.\right)$ at the equator where $\phi = 90°$. The increase in thickness below the equator is more rapid. At $\phi = 140°$ where the live load on the moving platforms, their weight, and that of their reinforced concrete supports are transmitted, the shell thickness is 15 in.—$\frac{1}{74}$ of the outside radius. Below this point the shell is stiffened by meridional ribs, its thickness rapidly increases and at the face of the support ring reaches 36 in.—that is, $\frac{1}{30}$ of the outside radius. The ratios of thickness to radius indicate that the membrane theory of shells, which assumes small thicknesses and uniform distribution of the strains in the cross section of the shell, is applicable to this structure except in the vicinity of the support ring. This assumption of uniform strains results in the disappearance of the bending and torsion moments perpendicular to the center surface of the shell and permits the calculation of the direct stresses, caused by axially symmetrical loadings, from equilibrium requirements.

The dead load and similarly distributed loads produce direct stresses in the direction of the meridians and parallel circles, causing small deformations of the shell. Rigorous investigation of spherical shells[6] indicate an upper limit of $\frac{\delta^2}{2a}$ for the magnitude of these deformations. For a shell thickness of $\delta = 4$ in. and a radius of $a = 90$ ft the largest eccentricity is thus 0.0074 in. The bending moments introduced by such deformations are negligibly small. The moments produced by the support conditions, however, are considerable. The membrane stress condition at the support ring is thoroughly disturbed resulting in large bending stresses in the direction of the meridians. It was specified that the outside surface of the shell should be a perfect sphere. For this reason the thickness of the shell increases inward and the middle surface, which bisects the thickness of the shell, deviates from a true spherical surface. The radii of curvature of this surface do not differ significantly from that of a sphere, except in the vicinity of the support ring where the change of its curvature is considerable and unfavorable. At points where the curvature of the middle surface changes suddenly, the change of the ring stresses is abrupt, and bending stresses are created in the direction of the meridians. By varying the shell thickness so that the middle surface is represented by a practically continuous function the abrupt changes in the ring stresses and the introduction of moments can be eliminated.

The dead and live loads were assumed to be symmetrical about the vertical axis of the sphere. The symmetry of the dead load and the smallness of the live load (about $\frac{1}{30}$ of the dead load) justified this assumption. The snow load was assumed to be 25 lb per sq ft at the top of the sphere, diminishing as the cosine of ϕ to zero at the equator. This assumption represents a total snow load about 60% larger than that used by the authors for the steel design, although that assumed by the authors was ample for any condition that would have occurred. The effect of this load, which is about $\frac{1}{20}$ of the dead load, is small in the lower part of the shell. The computation of the wind stresses due to the given pressure distribution could be only approximate. These stresses, as compared to the dead-load stresses, have significant values only at the upper part of the sphere where the shell thickness, determined by construction considerations, assures small unit stresses. The advantageous relation between the dead-load and wind-load stresses assures that the shell deformations produced by the unsymmetrical wind loads are small enough to make the bending moments introduced by these deformations negligible.

The maximum membrane stresses in the concrete produced by the combined dead, live, snow, and wind loads do not exceed 188 lb per sq in. tension and 224 lb per sq in. compression.

At $\phi = 155°$ the shell is joined to the support ring which rests on eight columns. This ring is compressed radially by the horizontal component of the meridian stresses. The abrupt change of the ring stresses at the support ring produces bending and shearing stresses perpendicular to the middle

[6] "Spannungen in Kugelschalen," by H. Reissner, *Müller Breslau Festschrift*, Kröner, Leipzig, 1912, p. 181.

surface of the shell. Temperature difference between the exposed shell and the inclosed support ring also has the same effect. Fig. 18(*g*), (*h*), and (*i*), shows the effect of the support ring on the shell. These stresses are due to dead, live, snow, and wind loads and 20° F temperature difference between the shell and the support ring. They were computed with consideration of the variation of the thickness of the shell.[7] Both vertical and horizontal components of the meridian stresses create bending stresses that are carried by the support ring in cooperation with the shell. The column reactions also effect additional stresses in the shell. The significant values of all of these stresses are localized in the vicinity of the support ring. They were determined by the application of theories developed primarily by Fr. Dischinger[8] and W. Flügge.[9] The maximum concrete unit stress produced by the combined effect of all loads and 20° F temperature difference between the shell and support ring is 574 lb per sq in. compression. To aid the distribution of the concentrated loads and stiffen the shell near the support, meridional ribs were arranged below $\phi = 140°$. Between the columns a shell 2.5 in. thick was suspended from the support ring. It has no statical purpose; it only completes the sphere.

The quantities of the structure are as follows:

Item	Concrete, in cu yd	Reinforcing steel, in lb
Shell	1,860	650,000
Ribs, framing and support ring	1,035	209,000
Suspended shell	29	6,000
Columns	56	25,000
Total	2,980	890,000

The writer believes that the costs given by the authors is high for Design No. 5. The construction of the lower part, up to $\phi = 120°$, where 70% of the quantities are located, does not differ from common reinforced concrete jobs, except that the falsework supporting the forms of this part should be kept in place until the concrete of the entire structure hardens. The building of the upper part does not involve considerably greater difficulties than the construction of the numerous large shells which have overcome, successfully, the competition of framed steel structures. The writer is inclined to believe that bids based on the working drawings of the concrete design would have more than justified the cost of extensive model testing.

The authors showed ingenuity in the analysis and great structural skill in the layout and design of the steel structure. This is demonstrated by the fact that this unusual structure could be both fabricated and erected without difficulties.

[7] "Über die Festigkeit Achsensymmetrischer Schalen," by J. Geckeler, *Forschungsarbeiten*, Heft 276, 1926, p. 19.

[8] "Die Rotationsschalen mit unsymmetrischer Form und Belastung," by Fr. Dischinger, *Der Bauingenieur*, Vol. 16, 1935, p. 393.

[9] "Die Statik und Dynamik der Schalen," by W. Flügge, Springer, Berlin, 1934, p. 43.

SHORTRIDGE HARDESTY,[10] M. AM. SOC. C. E., AND ALFRED HEDEFINE,[11] ASSOC. M. AM. SOC. C. E. (by letter).—The writers wish to express their appreciation to Mr. Balog for the extension and development of Design No. 5 in his discussion of their paper. Mr. Balog made the design of the reinforced concrete shell structure; and his design was then checked and included as one of the five designs submitted for consideration to the Construction Department of the Fair.

After studying the information obtained from a number of sources as to the advisability of building the concrete type and its probable cost, the figure that appears in the paper was determined. The writers do not believe this estimate of cost for the concrete design was unduly high. The adopted steel design was built at rather high unit prices because of the unprecedented problems involved; but the same factors would have operated in the case of the concrete design.

In the preliminary design studies of the Perisphere, a set of simultaneous equations was written expressing the elastic displacements of meridians and girts at their intersections. This process was discarded early, because it proved impracticable and too complex. The structure, as stated in the paper, was extremely sensitive to applied loads. Consequently, any load changes (and there were many), even though relatively small, involved large variations in the coefficients of the simultaneous equations and made their solutions laborious and time consuming.

The writers agree with Mr. Balog that it is regrettable that no stress measurements were made on the structure as built. On several occasions attempts were made to arouse sufficient interest to secure money for this feature, but they were unsuccessful.

The structures, comprising 3,200 tons of steel and four acres of covering, displays, and equipment, were completely demolished in the spring of 1941. In the main, the procedure followed in demolition was the reverse of that used during construction for both the Trylon and the Perisphere.

It was hoped that some discussion would be presented relative to the possibility of designing a sphere of this size by making a steel shell self-supporting without the use of any framing, which would have been a very complicated problem since there were really two spheres, one inside the other. Such a type of construction was suggested in the early stages of the design, but was discarded because of the practical problems of fabrication and construction involved.

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